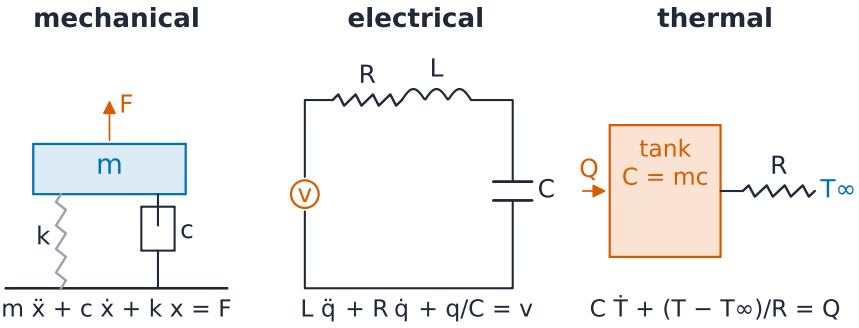
# System Dynamics and Control

Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

# Closing the loop

- The last course: a **dc motor** turns a ball valve through 50:1; a controller sets it from the loop temperature.
- Every simulated response is checked against an analytical checkpoint.
- Write the model's assumptions before its equations.

# One equation form, three domains




Reservoir 200 L: $C = \mathbf{836}$ kJ/K, $R = 0.01$ K/W, $\tau = RC = \mathbf{2.3}$ h; 1 kW raises it $\mathbf{10}$ K.

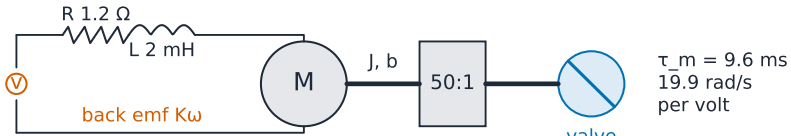
# The mount in state space

1. **States** $x_1 = x$, $x_2 = \dot{x}$; $m = 120$, $c = 910$, $k = 300\,000$.
2. **Matrices** $\mathbf{A} = [0, 1;\ -2500, -7.58]$, $\mathbf{B} = [0;\ 1/120]$.
3. **Eigenvalues** $-3.79 \pm 49.86j$ - the decay record's $\zeta\omega_n$ and $\omega_d$.

## Decision

One matrix form now serves the mount, the motor and the loop.

# The valve actuator




$K = 0.05$: **19.9** rad/s per volt, $\tau_m = JR/K^2 = \mathbf{9.6}$ ms, $\tau_e = 1.7$ ms; stall 10 A, 0.5 N · m.

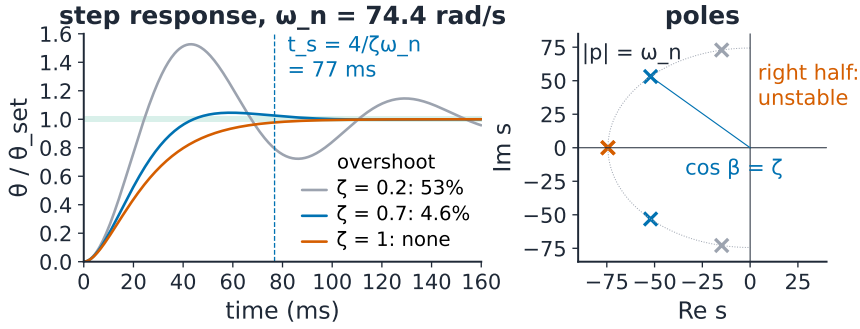
# Through the gearbox

1. **Valve gain** $19.9/50 = \mathbf{0.398}$ rad/s per volt; 12 V $\to$ 4.78 rad/s, 90° in $\geq$ 0.33 s.
2. **Load** 5 N · m at the stem $\to$ 0.125 N · m at the motor $\to$ $\mathbf{3.0}$ V.
3. **Current limit** 5 A $\to$ 0.25 N · m $\to$ $\mathbf{10}$ N · m at the valve, twice the load.

## Decision

The model is valid while voltage and current stay inside their limits.

# Poles and the step response




$K_p = \mathbf{134}$ V/rad: $\omega_n = \mathbf{74.4}$ rad/s, $\zeta = 0.7$, **overshoot** $\mathbf{4.6}\%$, $t_s = \mathbf{77}$ ms.

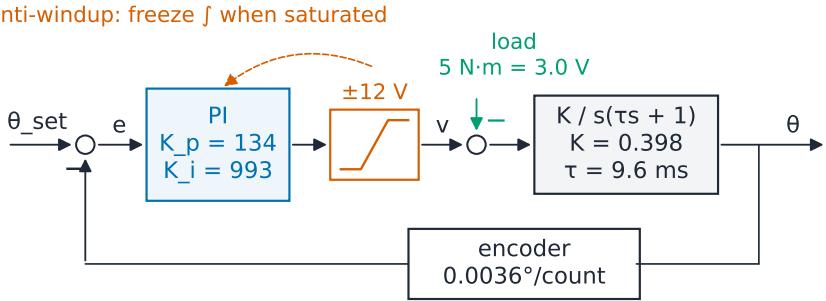
# Margins from the frequency response

1. **Open loop** $L = 53.1/[j\omega(0.0096j\omega + 1)]$.
2. **Crossover** $|L| = 1$ at $\omega_c = \mathbf{48.2}$ rad/s; phase $-114.8°$.
3. **Margin** $PM = \mathbf{65.2}°$; $\zeta \approx PM/100 = 0.65$, close to the 0.7 designed.

**Decision**

Gain alone cannot destabilise this loop - delay can.

# The position loop




P alone stops $3.0/134$ rad $=$ $\mathbf{1.29}^\circ$ short under load; $K_i = K_p\omega_n/10 = \mathbf{993}$ removes it.

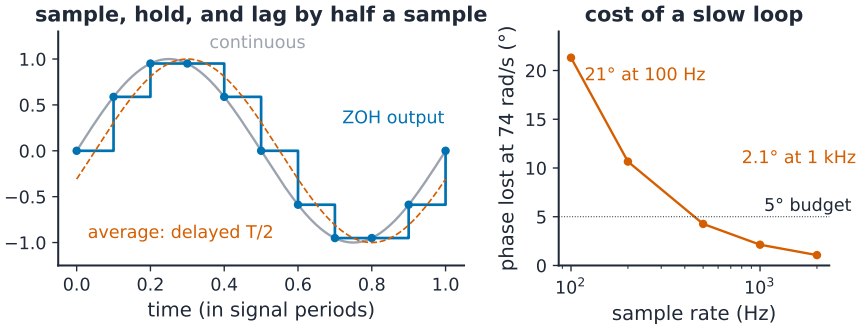
# Saturation and windup

1. **Demand** 90° step $\times$ 134 V/rad $=$ **210** V asked of a 12 V driver.
2. **Rate limit** $0.398 \times (12 - 3.0) = 3.58$ rad/s under load, for about 0.4 s.
3. **Windup** Integral grows unused; valve carries on to **159**°, in band only at 1.32 s.
4. **Clamp** Hold the integral while saturated $\rightarrow$ no **overshoot**, in band at 0.52 s.

## Decision

Test large steps, not only the small ones the linear design predicts.

# Sampling costs phase




Hold lag $\omega T/2$ at 74.4 rad/s: $\mathbf{2.1}°$ at 1 kHz, $\mathbf{21}°$ at 100 Hz. Choose 1 kHz.

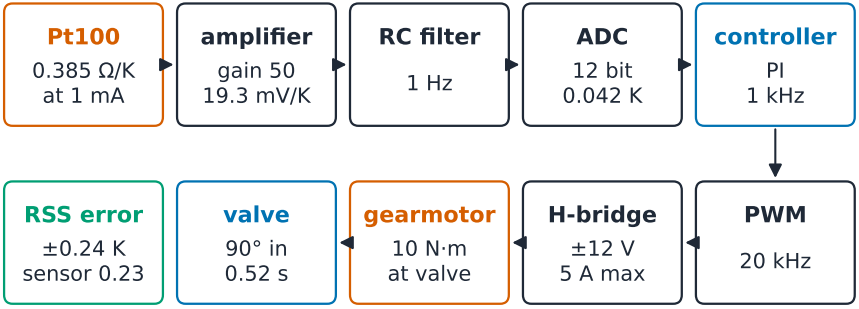
# Tustin, quantisation, latency

1. **Integral** $I_k = I_{k-1} + \frac{T}{2}(e_k + e_{k-1})$; $u_k = K_p e_k + K_i I_k$.
2. **Encoder** 2000 counts/rev $\times$ 50 $\rightarrow$ $\mathbf{0.0036}^\circ$ per count at the valve.
3. **Speed** 1 count per ms $= \mathbf{3.14}$ rad/s at the motor - too coarse for derivative action.
4. **Latency** 0.2 ms computation $\rightarrow$ 0.6° more lag at **crossover**.

## Decision

Position resolution is ample; speed estimates and delay are the real limits.

# The signal chain




Pt100 at 1 mA $\times$ 50 $=$ 19.25 mV/K; 12-bit **ADC** step $\mathbf{0.042}$ K; RSS budget $\pm\mathbf{0.24}$ K at 40 °C.

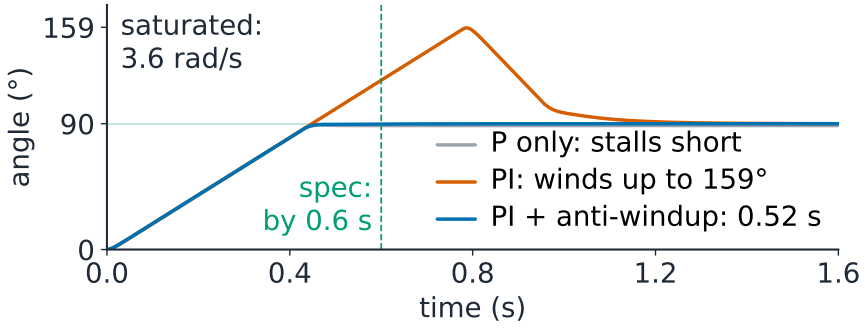
# Error budget and actuator limits

1. **Sensor** Class A, $\pm(0.15 + 0.002\,T) = \pm 0.23$ K at 40 °C.
2. **Others ADC** 0.012, gain 0.04, self-heating 0.05 K.
3. **RSS** $\sqrt{0.23^2 + 0.012^2 + 0.04^2 + 0.05^2} = \pm\mathbf{0.24}$ K.
4. **Actuator** 12 V $\rightarrow \leq$ 4.78 rad/s; 5 A $\rightarrow$ 10 N $\cdot$ m; about 30 W at the motor.

**Decision**

A better ADC would change nothing; a better sensor would.

# The project, simulated




Spec $\pm 0.5°$ by 0.6 s, 5 N · m load: P fails (1.29° short), PI fails (159°), PI + **anti-windup** passes at $\mathbf{0.52}$ s.

# Reporting the limits

1. **No load** A 5° step overshoots $18\%$ - the integral zero, normally hidden by the load.
2. **Margin** 0.52 s against 0.6 s; at 11 V only 0.58 s.
3. **Untested** Encoder faults, sticking, copper $R$ $+0.4\%/\mathrm{K}$.

**Decision**

Passing is a result with a margin and limits, never just a tick.

# Check yourself

- The tank and the motor? ($\tau$ 2.3 h, 10 K; 19.9 rad/s/V, 9.6 ms, 3.0 V holding.)
- The P loop? ($K_p$ 134, $\omega_n$ 74.4, 4.6%, 77 ms; PM 65.2°; 1.29° error.)
- The PI loop? ($K_i$ 993; 159° with windup; 0.52 s with clamping.)
- The digital side and the sensor? (2.1° at 1 kHz, 21° at 100 Hz; 0.042 K/LSB, $\pm$0.24 K.)