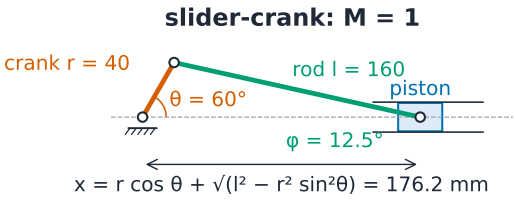
Theory of Machines and Vibrations

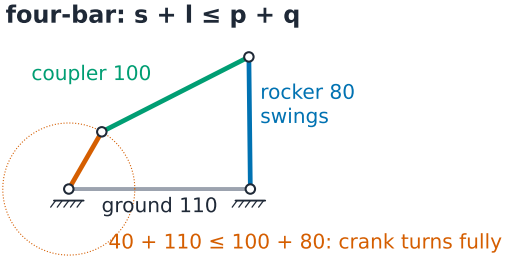
Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

How mechanisms move, how machines shake

- Two machines: the dosing pump (slider-crank, 300 rpm) and the main pump (120 kg, 1440 rpm) on mounts.
- Every answer is checked by a second, independent method.
- Units 11.1-11.3 move and balance; 11.4-11.6 vibrate.

Slider-crank and four-bar





$r = 40$, $l = 160$, $\theta = 60°$: $\phi = \mathbf{12.5}°$, $x = \mathbf{176.2}$ mm, stroke 80 mm; $M = 3(n-1) - 2j_1 = \mathbf{1}$.

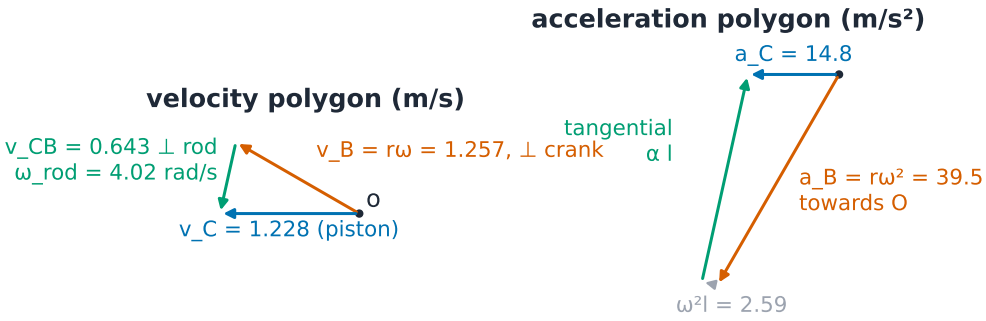
Counting and inverting

1. **Four-bar** 40/100/80/110, $\theta_2 = 60^\circ \to \theta_3 = \mathbf{27.0}^\circ$, $\theta_4 = \mathbf{90.6}^\circ$.
2. **Type** Shortest link as crank $\to$ crank-rocker; ground the coupler $\to$ double-rocker.
3. **Five-bar** 5 **links**, 5 pins $\to M = 3 \times 4 - 2 \times 5 = \mathbf{2}$ inputs.

Decision

Count before you calculate; the count says how many motors you need.

Velocity and acceleration polygons



300 rpm, 60°: $v_B = \mathbf{1.257}$, $v_C = \mathbf{1.228}$ m/s, $\omega_{rod} = \mathbf{4.02}$ rad/s; $a_B = 39.5$, $a_C = \mathbf{14.8}$ m/s^2.

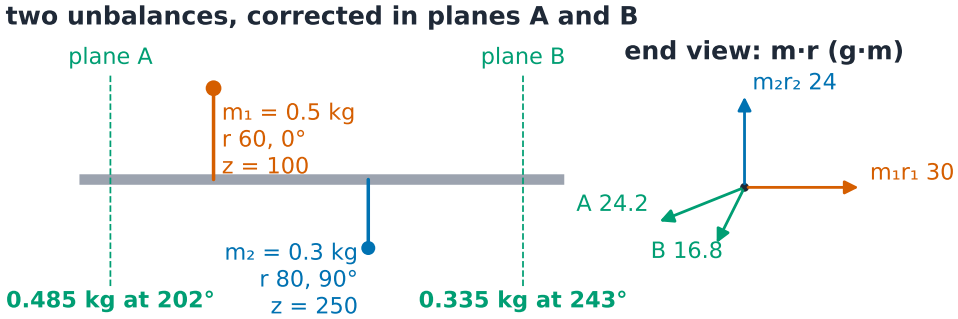
Two checks and Coriolis

1. **Instant centre** Rod about I: $1.257/0.312 = 1.228/0.305 = \mathbf{4.02}$ rad/s.
2. **Formula** $a_C \approx r\omega^2(\cos\theta + \cos 2\theta/n) = 14.80$ m/s^2, $n = 4$.
3. **Coriolis** Block at 0.6 m/s on an arm at 5 rad/s: $2\omega v_{rel} = \mathbf{6.0}$ m/s^2.

Decision

Two methods that agree are worth more than one careful one.

Balancing in two planes



Corrections at 50 mm: $\mathbf{0.485}$ kg at $\mathbf{202}°$ in A, $\mathbf{0.335}$ kg at $\mathbf{243}°$ in B; 873 N of shaking removed.

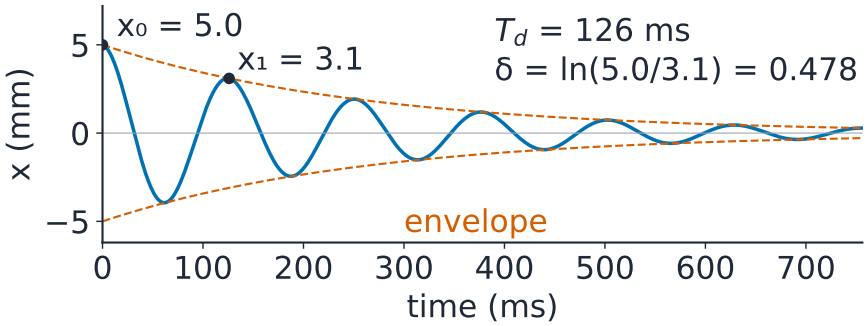
The reciprocating mass

1. **Primary** 0.8 kg, 40 mm, 31.42 rad/s $\rightarrow$ $mr\omega^2 = \mathbf{31.6}$ N, once per turn.
2. **Secondary** $mr\omega^2/n = \mathbf{7.9}$ N, twice per turn.
3. **Half balance** 15.8 N left along the stroke, 15.8 N added across it.

Decision

Partial balance trades a large force in one direction for a smaller one that turns.

Free decay and damping



$k = 300$ kN/m, $m = 120$ kg: $f_n = \mathbf{7.96}$ Hz. Peaks 5.0 → 3.1 mm: $\delta = 0.478$, $\zeta = \mathbf{0.076}$.

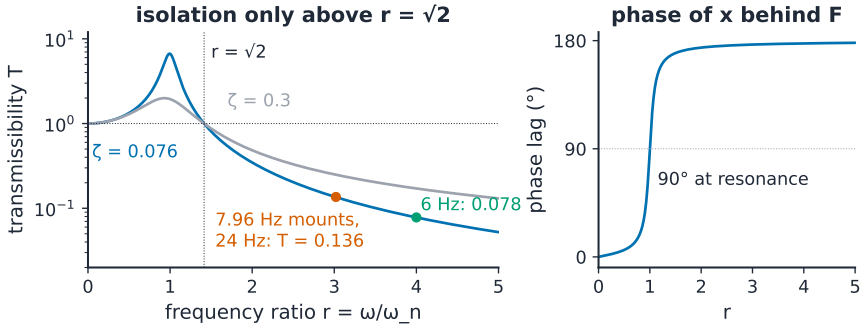
Torsion between gear and impeller

1. **Stiffness** Ø30, 0.6 m, $G = 80$ GPa $\rightarrow$ $k_t = GJ_p/L = \mathbf{10.6}$ kN $\cdot$ m/rad.
2. **Inertias** Gear $J_1 = 0.64$, impeller $J_2 = 0.12$ kg $\cdot$ m^2.
3. **Frequency** $\omega = \sqrt{k_t(J_1 + J_2)/J_1 J_2} = \mathbf{324}$ rad/s $= 51.6$ Hz, node 95 mm from the gear.

Decision

3090 rpm against 180 rpm running - no torsional **resonance**.

Transmissibility and phase



24 Hz on 7.96 Hz mounts: $r = \mathbf{3.02}$, $T = \mathbf{0.136}$ - 3.7 N of the 27.3 N reaches the floor.

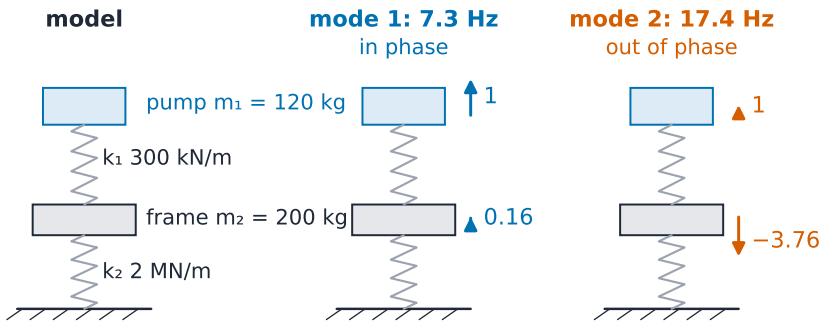
Designing for 90% isolation

1. **Target** $T \le 0.1$ at $\zeta = 0.076 \to r \ge 3.40 \to f_n \le 7.06$ Hz.
2. **Choose** $f_n = 6$ Hz $\to$ $k = \mathbf{170.5}$ kN/m, four mounts of 42.6, deflection 6.9 mm.
3. **Result** $r = 4.0$, $T = \mathbf{0.078}$, 2.1 N to the floor.

Decision

Soft enough to isolate, stiff enough that the pipes do not pull the pump about.

Two masses, two modes



Pump 120 kg on 300 kN/m, frame 200 kg on 2 MN/m: $\mathbf{7.30}$ Hz in phase (1, 0.16), $\mathbf{17.4}$ Hz out of phase (1, −3.76).

Response at running speed

1. **Matrices** $\mathbf{M} = \mathrm{diag}(120, 200)$; $\mathbf{K} = [300, -300; -300, 2300]$ kN/m.
2. **Solve** $(\mathbf{K} - \omega^2\mathbf{M})\mathbf{X} = (27.3, 0)$ at 150.8 rad/s.
3. **Result** Pump $\mathbf{11.4}$ µm, frame $\mathbf{1.5}$ µm, floor $k_2 X_2 = \mathbf{3.0}$ N.

Decision

24 Hz lies above both modes, so the two-mass model confirms the single-mass estimate.

Check yourself

- The crank and the rod? (176.2 mm, 12.5°; 1.228 m/s, 4.02 rad/s, 14.8 m/s^2.)
- The shaft and the piston? (0.485 kg at 202°, 0.335 kg at 243°; 31.6 N and 7.9 N.)
- The mounts? (7.96 Hz, ζ 0.076; T 0.136 at 24 Hz; 6 Hz mounts give 0.078.)
- The torsion and the frame? (51.6 Hz; 7.30 and 17.4 Hz, floor force 3.0 N.)