Solutions

2.1

- $\omega_n = \sqrt{25/4} = 2.5$ rad/s; $\zeta = 4/(2\sqrt{100}) = 0.2$ - underdamped.
- $\omega_d = 2.5\sqrt{0.96} = 2.4495$ rad/s.

2.2

- Envelope $e^{-\zeta\omega_n t} = e^{-0.5t}$: at 2 s, $e^{-1} = 0.368$ mm-scale.
- 1% at $0.5t = 4.6$: $t = 9.2$ s.

2.3

- $M(1) = 1/(2 \times 0.2) = 2.5$.
- Drive $\leq 5/2.5 = 2$ mm.

2.4

- Substitute x, $\dot{x}$, $\ddot{x}$ from the claimed formula into the ODE: residual at machine precision.
- Check the initial conditions match, and the first-extremum value against $-e^{-\zeta\pi/\sqrt{1-\zeta^2}}$.

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