Solutions

2.1

- $C = mc = 150 \times 4180 = \mathbf{627}$ kJ/K.
- $R = 1/UA = \mathbf{0.02}$ K/W.
- $\tau = RC = 0.02 \times 627\,000 = \mathbf{12\,540}$ s $= \mathbf{3.48}$ h.

2.2

- Final rise $= QR = 2000 \times 0.02 = \mathbf{40}$ K.
- $\Delta T = QR(1 - e^{-t/\tau}) = 40(1 - e^{-7200/12\,540}) = \mathbf{17.5}$ K.
- $30 = 40(1 - e^{-t/\tau})$ gives $t = \tau \ln 4 = \mathbf{4.83}$ h.

2.3

- The water is well mixed, so it has one temperature (or: UA is constant; the wall stores no heat).

3.1

- (a) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -400 & -8 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 \\ 0.02 \end{bmatrix}$.
- $\det(\lambda\mathbf{I} - \mathbf{A}) = \lambda^2 + 8\lambda + 400 = 0$, so $\lambda = -\mathbf{4} \pm \mathbf{19.6j}$.
- $\omega_n = |\lambda| = \sqrt{400} = \mathbf{20}$ rad/s; $\zeta = 4/20 = \mathbf{0.2}$.
- (b) $L\ddot{q} + R\dot{q} + q/C = V$: inductance $\leftrightarrow$ mass, resistance $\leftrightarrow$ damper, $1/C \leftrightarrow$ spring stiffness, voltage $\leftrightarrow$ force.
- $\omega_n = 1/\sqrt{LC} = 1/\sqrt{0.5 \times 500 \times 10^{-6}} = \mathbf{63.25}$ rad/s.
- $\zeta = (R/2)\sqrt{C/L} = 20\sqrt{0.001} = \mathbf{0.632}$.

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