## Solutions

**2.1**

- $\mathbf{M} = \mathrm{diag}(100, 150)$ kg.
- $\mathbf{K} = \begin{bmatrix} 200 & -200 \\ -200 & 1700 \end{bmatrix}$ kN/m.

**2.2**

- $(k_1 - \omega^2 m_1)(k_1 + k_2 - \omega^2 m_2) - k_1^2 = 0$.
- Expanding: $m_1 m_2 \omega^4 - [m_1(k_1 + k_2) + m_2 k_1]\omega^2 + k_1 k_2 = 0$, that is $15\,000\omega^4 - 2.0 \times 10^8 \omega^2 + 3.0 \times 10^{11} = 0$.
- $\omega^2 = 1722.5$ or $11\,611$.
- $\omega_1 = \mathbf{41.50}$ rad/s (**6.61** Hz); $\omega_2 = \mathbf{107.75}$ rad/s (**17.15** Hz).

**3.1**

- (a) $u_2 = 1 - \omega^2 m_1 / k_1 = 1 - \omega^2/2000$.
- Mode 1: $u_2 = 1 - \omega_1^2 m_1/k_1 = 1 - 1722.5/2000 = \mathbf{0.139}$: both move together, the mounts flex.
- Mode 2: $u_2 = 1 - \omega_2^2 m_1/k_1 = 1 - 11\,611/2000 = \mathbf{-4.805}$: the frame bounces against the fan on its stiff feet.
- (b) $\omega^2 = 24\,674$: $\mathbf{K} - \omega^2\mathbf{M} = \begin{bmatrix} -2.267 & -0.2 \\ -0.2 & -2.001 \end{bmatrix} \times 10^6$ N/m.
- Solving: $X_1 = \mathbf{-22.0}$ µm, $X_2 = \mathbf{2.19}$ µm; the fan moves in antiphase with the force (above both modes).
- Floor force $= k_2 X_2 = 1.5 \times 10^6 \times 2.19 \times 10^{-6} = \mathbf{3.3}$ N.
- (c) Alone on $k_1$: $X = F_0/(k_1 - \omega^2 m_1) = -21.8$ µm, and the floor would feel $k_1 X = 4.4$ N; the frame adds a little isolation.
- With mode 2 at 25 Hz, the running speed would excite it: the frame amplitude and floor force would rise sharply, so every mode must be checked, not only the first.



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