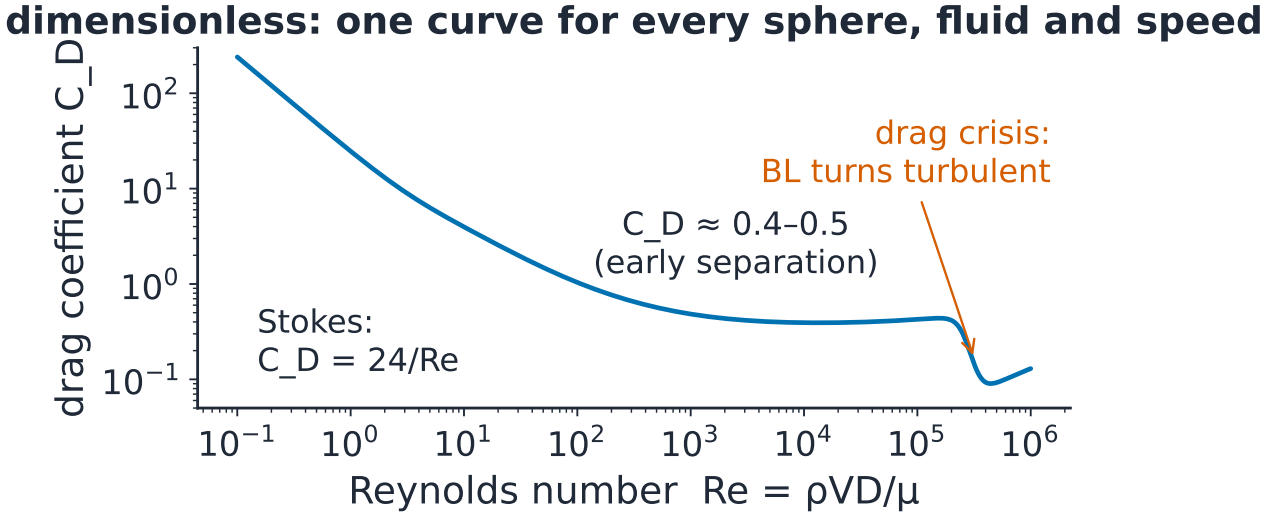
Exercise walk-through

7.4
Dimensional analysis

University Mechanical Engineering

Dimensional analysis: one curve for every sphere



Five variables F, ρ, V, D, μ in three dimensions give two groups: $C_D = F/(\frac{1}{2}\rho V^2 A) = \phi(Re)$, with $Re = VD/\nu$.

The model test in a water tunnel

1. **Prototype** Pod 0.5 m at 20 m/s in air: $Re = 6.60 \times 10^5$.
2. **Model** Half scale in water: $V_m = 20 \times 2 \times (1.004 \times 10^{-6}/1.516 \times 10^{-5}) = \mathbf{2.65}$ m/s.
3. **Scale back** $F_m = 30$ N: $F_p = F_m(\rho_p/\rho_m)(V_p/V_m)^2(L_p/L_m)^2 = \mathbf{8.25}$ N.
4. **Pipe regime** Loop: $Re = 2.39 \times 0.08/1.004 \times 10^{-6} = \mathbf{1.90 \times 10^5}$ - turbulent.

Exercises 2.1·2.2

2.1 The drag F on a smooth sphere depends on ρ, V, D and μ. How many dimensionless groups govern it, and what are they? [3]

2.2 Water flows at 1.2 m/s in a 50 mm pipe. Find Re (nearest hundred) and say whether the flow is laminar or turbulent. [3]

Exercises 2.1·2.2

2.1 The drag F on a smooth sphere depends on ρ, V, D and μ. How many dimensionless groups govern it, and what are they? [3]

2.1

- Five variables (F, ρ, V, D, μ) in three dimensions (M, L, T): $5 - 3 = \mathbf{2}$ groups.
- $F/(\rho V^2 D^2)$, a drag coefficient, and $\rho V D/\mu = Re$.

2.2 Water flows at 1.2 m/s in a 50 mm pipe. Find Re (nearest hundred) and say whether the flow is laminar or turbulent. [3]

2.2

- $Re = VD/\nu = 1.2 \times 0.05/(1.004 \times 10^{-6}) = \mathbf{59\,800}$.
- Far above 4000: **turbulent**.

Exercises 2.3·3.1

2.3 A car 4.0 m long travels at 25 m/s. A 1:5 model is tested in a water tunnel. Find the tunnel speed needed for dynamic similarity (two decimals, m/s). [3]

3.1 At the speed of 2.3 the model's drag is 900 N. Find the full-size drag (nearest newton) and the car's drag coefficient if its frontal area is 2.2 m^2 (two decimals). Why would matching Re in a wind tunnel at 1:5 be impractical? [6]

Exercises 2.3·3.1

2.3 A car 4.0 m long travels at 25 m/s. A 1:5 model is tested in a water tunnel. Find the tunnel speed needed for dynamic similarity (two decimals, m/s). [3]

2.3

- Equal Re: $V_m = V_p(L_p/L_m)(\nu_m/\nu_p) = 25 \times 5 \times (1.004 \times 10^{-6}/1.516 \times 10^{-5}) = \mathbf{8.28}$ m/s.

3.1 At the speed of 2.3 the model's drag is 900 N. Find the full-size drag (nearest newton) and the car's drag coefficient if its frontal area is 2.2 m^2 (two decimals). Why would matching Re in a wind tunnel at 1:5 be impractical? [6]

3.1

- Equal C_D: $F_p = F_m(\rho_p/\rho_m)(V_p/V_m)^2(L_p/L_m)^2 = 900 \times (1.204/998) \times (25/8.28)^2 \times 5^2 = \mathbf{248}$ N.
- $C_D = F/(\tfrac{1}{2}\rho V^2 A) = 248/(0.5 \times 1.204 \times 25^2 \times 2.2) = \mathbf{0.30}$.
- In air the model would need $25 \times 5 = 125$ m/s, a third of the speed of sound, so compressibility would spoil the similarity.