

Exercise walk-through

6.4

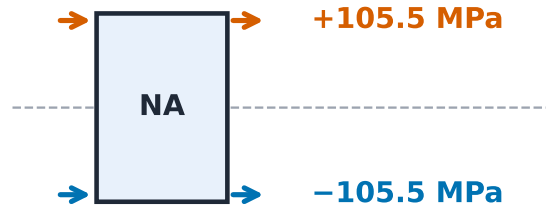
Beam stresses and deflection

University Mechanical Engineering

Beam stresses: linear and parabolic

bending is linear in depth, shear is parabolic

40 × 80 mm at $M = 4.5 \text{ kN}\cdot\text{m}$

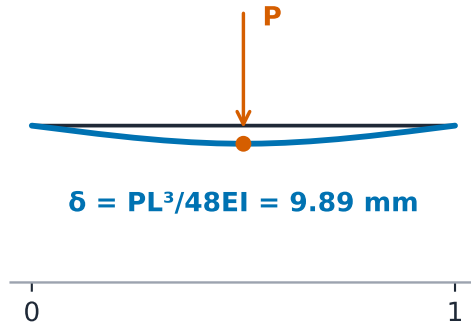


On the 40 × 80 mm arm: $\sigma = Mc/I = 105.5 \text{ MPa}$ (or M/S with $S = 42.7 \times 10^{-6} \text{ m}^3$), and $\tau_{max} = 1.5V/A = 1.41 \text{ MPa}$ on the neutral axis.

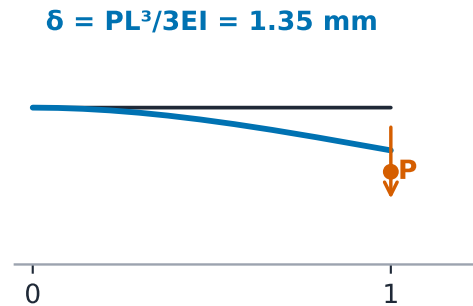
Deflection: stiffness is a spec

two standard cases, two cubes of L

mid-span, central load



cantilever, tip load



Central 6 kN on 3 m: $\delta = PL^3/48EI = 9.89 \text{ mm}$. Cantilever 1.2 m with 800 N: $\delta = PL^3/3EI = 1.35 \text{ mm}$ - double the depth, eighth the sag.

Exercises 2.1·2.3

2.1 Simply supported over 4 m with a central $P = 8$ kN: the maximum bending stress (one decimal, MPa) and the section modulus (one decimal, 10^{-6} m³). [4]

2.2 The same beam at its maximum shear force: the maximum transverse shear stress (one decimal, MPa). [3]

2.3 The same beam and load: the mid-span deflection (one decimal, mm). [4]

Exercises 2.1-2.3

2.1 Simply supported over 4 m with a central $P = 8$ kN: the maximum bending stress (one decimal, MPa) and the section modulus (one decimal, 10^{-6} m³). [4]

2.1

- $M_{max} = PL/4 = \mathbf{8.0}$ kN · m; $\sigma = Mc/I = 8000 \times 0.05 / (4.167 \times 10^{-6}) = \mathbf{96.0}$ MPa.
- $S = I/c = 4.167 \times 10^{-6} / 0.05 = \mathbf{83.3 \times 10^{-6}}$ m³.

2.2 The same beam at its maximum shear force: the maximum transverse shear stress (one decimal, MPa). [3]

2.2

- $V_{max} = P/2 = 4$ kN; $\tau_{max} = 1.5V/A = 1.5 \times 4000 / 0.005 = \mathbf{1.2}$ MPa.

2.3 The same beam and load: the mid-span deflection (one decimal, mm). [4]

2.3

- $EI = 200 \times 10^9 \times 4.167 \times 10^{-6} = 8.333 \times 10^5$ N m².
- $\delta = PL^3 / (48EI) = 512\,000 / (4.000 \times 10^7) = \mathbf{12.8}$ mm.

Exercise 2.4

2.4 The same section as a 1.5 m cantilever with $P = 1.2$ kN at the tip: the tip deflection (two decimals, mm), and the single cheapest change that would stiffen it. [3]

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2.4

- $\delta = PL^3/(3EI) = 4050/(2.500 \times 10^6) = \mathbf{1.62}$ mm.
- Increase the section depth: I carries h^3 - double the depth, one eighth the sag.