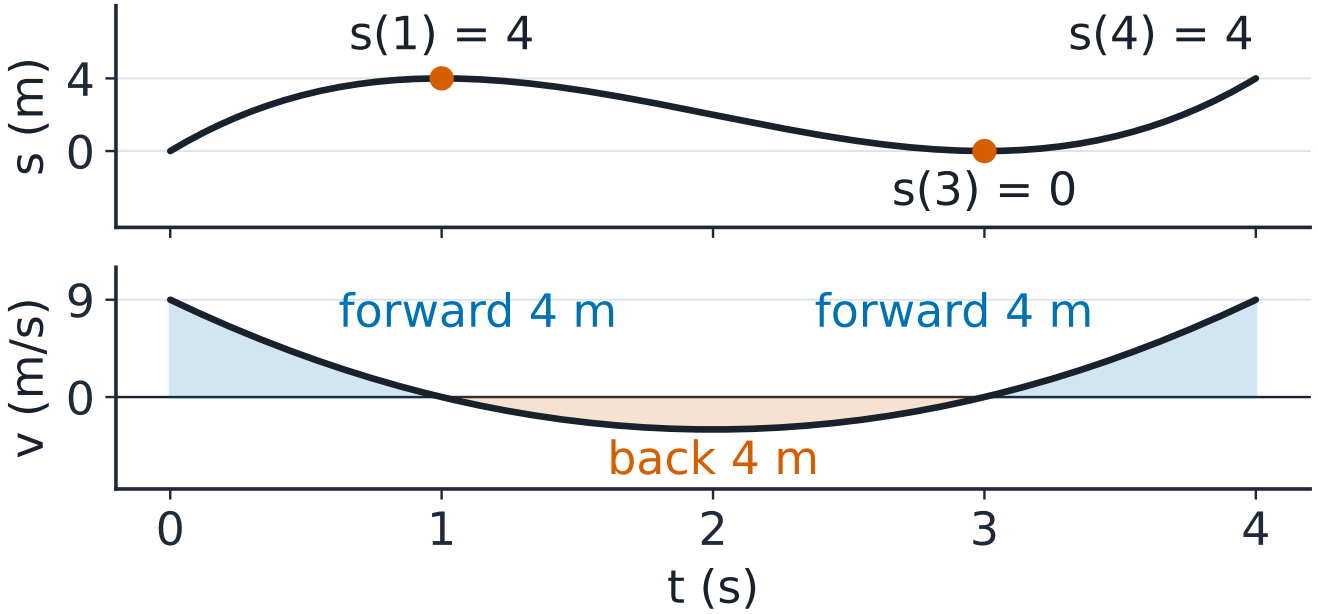
Exercise walk-through

2.4
Numerical methods

University Mechanical Engineering

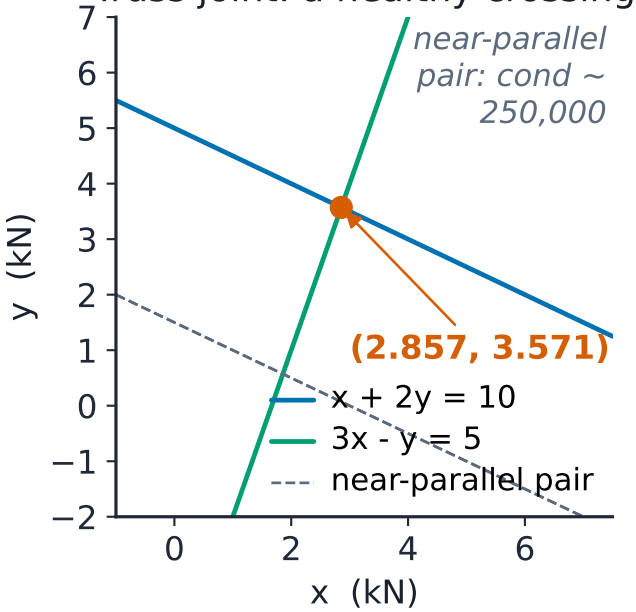
Calculus: the trace is a dictionary



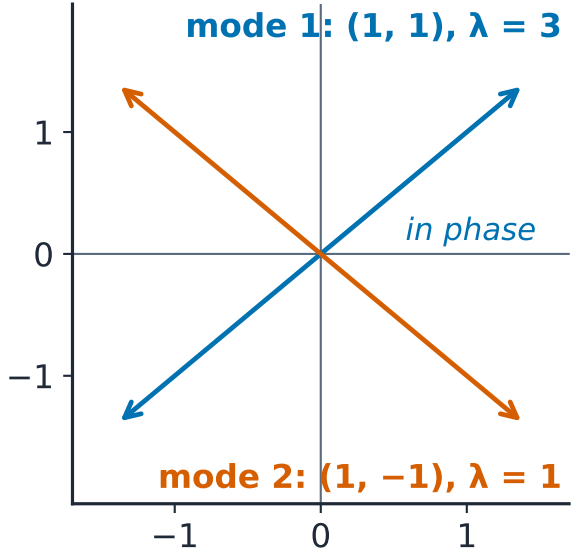
Displacement 4 m but distance **12 m**: v changes sign at t = 1 and 3.

Linear algebra: the condition number prices trust

Truss joint: a healthy crossing



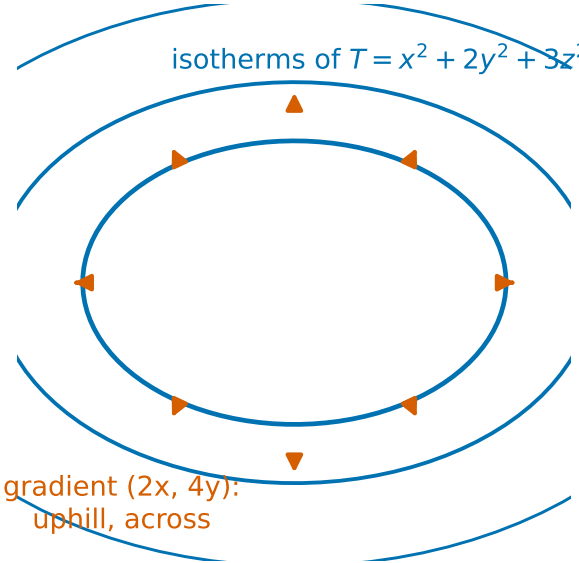
The rig matrix: two mode shapes



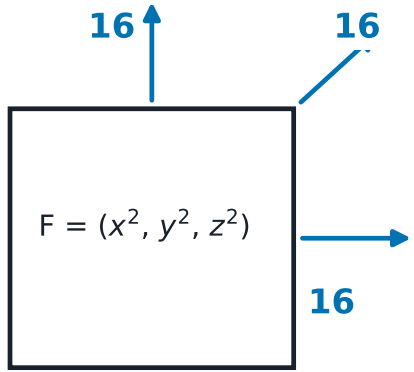
Truss lines cross at **(2.857, 3.571)** kN; rig modes (1, 1) and (1, −1).

Vector calculus: count what leaves

Fields: T rises at 7.483 K/m; q = -40 grad T



Flux 48, counted two ways



surface flux: 16 + 16 + 16 = 48

divergence: 2(x+y+z) dV = 48

∇T gives **7.483** K/m; **flux 48** by surface = **48** by **divergence**.

Exercises 2.1·2.2

2.1 From the two trapezoid errors, the order, shown as a ratio. [2]

2.2 The predicted trapezoid error at $n = 16$ (two significant figures), and the Simpson order from its own pair. [4]

Exercises 2.1·2.2

2.1 From the two trapezoid errors, the order, shown as a ratio. [2]

2.1

- $8.94/2.24 = 3.99 \approx 4$ per halving: order 2.

2.2 The predicted trapezoid error at $n = 16$ (two significant figures), and the Simpson order from its own pair. [4]

2.2

- Trapezoid at 16: $2.24 \times 10^{-3}/4 = 5.6 \times 10^{-4}$.
- Simpson $3.70 \times 10^{-5} \to 2.33 \times 10^{-6}$: ratio $15.9 \approx 16$: order 4.

Exercises 2.3·2.4

2.3 A budget of 10^{-6}: the n Simpson needs (power of two), and the n the trapezoid would need to the nearest power of ten. [4]

2.4 Why does the Simpson error not keep falling forever? Name the mechanism and its per-step scale. [4]

Exercises 2.3·2.4

2.3 A budget of 10^{-6}: the n Simpson needs (power of two), and the n the trapezoid would need to the nearest power of ten. [4]

2.3

- Simpson from $n = 8$ needs $2.33 \times 10^{-6}/16 < 10^{-6}$: $n = 16$ suffices.
- Trapezoid from $n = 8$: $2.24 \times 10^{-3}/4^p < 10^{-6}$ needs $p \approx 5.6$ halvings - n near $2^{14} \approx 16{,}000$.

2.4 Why does the Simpson error not keep falling forever? Name the mechanism and its per-step scale. [4]

2.4

- Round-off: each panel adds floating-point noise of order 10^{-16}.
- Past $\sim 10^5$ panels the accumulated round-off ($\sim 10^{-10}$) exceeds truncation - the U-curve floor.