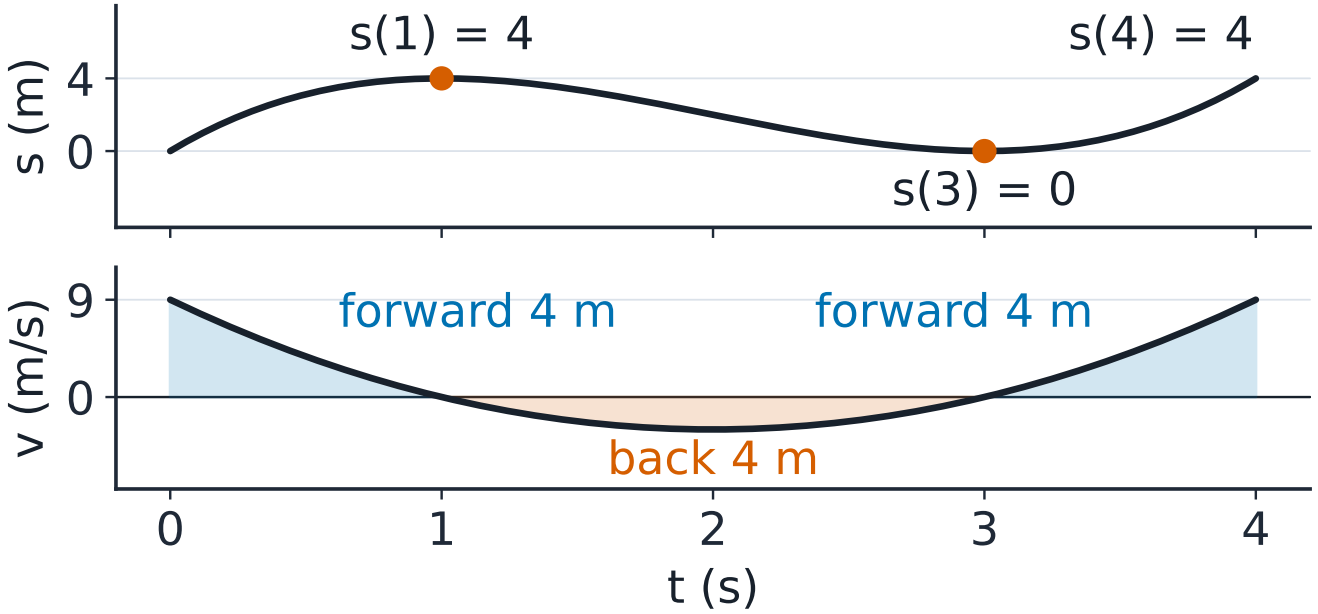
Exercise walk-through

2.3 Differential equations

University Mechanical Engineering

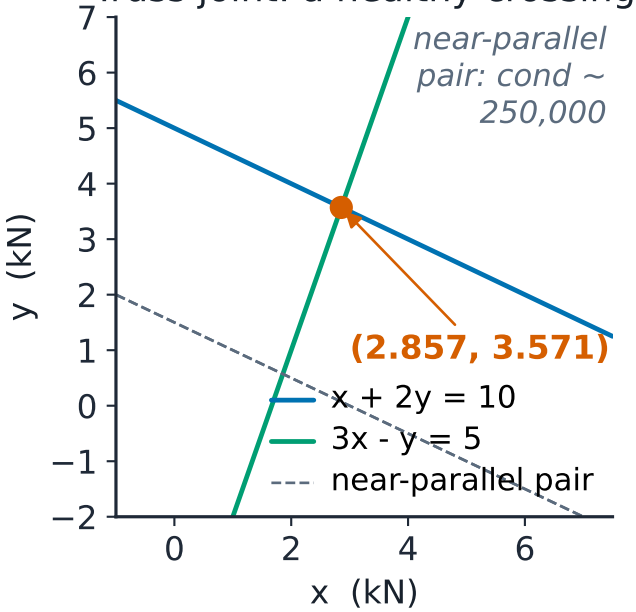
Calculus: the trace is a dictionary



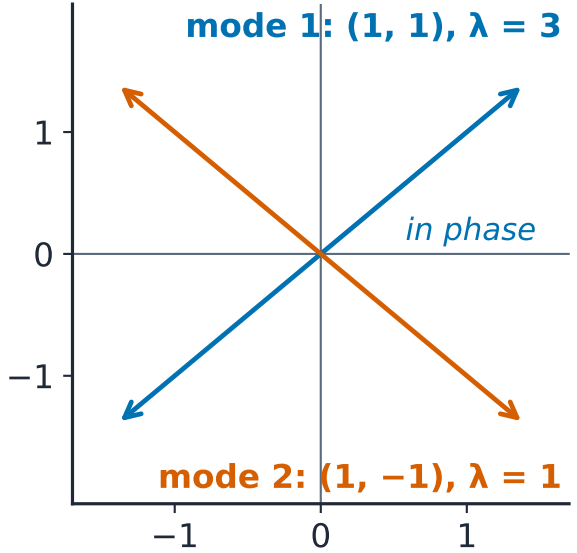
Displacement 4 m but distance **12 m**: v changes sign at t = 1 and 3.

Linear algebra: the condition number prices trust

Truss joint: a healthy crossing



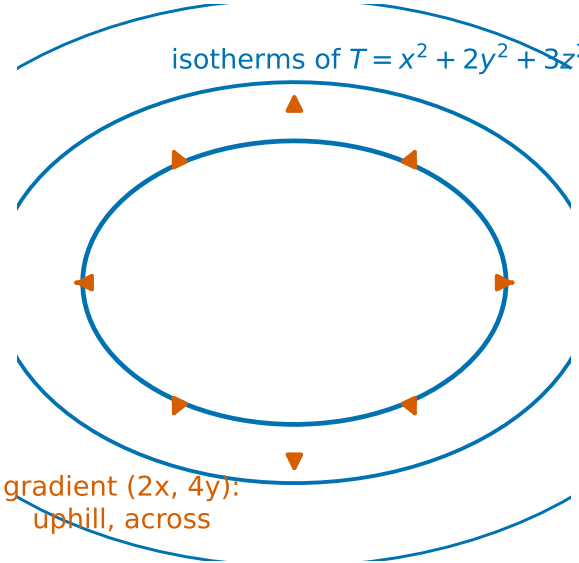
The rig matrix: two mode shapes



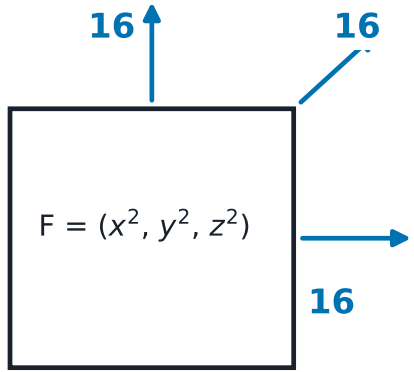
Truss lines cross at **(2.857, 3.571)** kN; rig modes (1, 1) and (1, −1).

Vector calculus: count what leaves

Fields: T rises at 7.483 K/m; q = -40 grad T



Flux 48, counted two ways



surface flux: 16 + 16 + 16 = 48

divergence: 2(x+y+z) dV = 48

∇T gives **7.483** K/m; **flux 48** by surface = **48** by **divergence**.

Exercises 2.1·2.2

2.1 ω_n, ζ, the response family, and ω_d (four decimals). [4]

2.2 From rest at $x_0 = 1$ mm: the envelope's value at $t = 2$ s, and the time to decay to 1% of the start. [3]

Exercises 2.1·2.2

2.1 ω_n, ζ, the response family, and ω_d (four decimals). [4]

2.1

- $\omega_n = \sqrt{25/4} = 2.5$ rad/s; $\zeta = 4/(2\sqrt{100}) = 0.2$ - underdamped.
- $\omega_d = 2.5\sqrt{0.96} = 2.4495$ rad/s.

2.2 From rest at $x_0 = 1$ mm: the envelope's value at $t = 2$ s, and the time to decay to 1% of the start. [3]

2.2

- Envelope $e^{-\zeta\omega_n t} = e^{-0.5t}$: at 2 s, $e^{-1} = 0.368$ mm-scale.
- 1% at $0.5t = 4.6$: $t = 9.2$ s.

Exercises 2.3·2.4

2.3 The floor drives the rig at $r = 1$. The magnification, and the drive amplitude if the till may see at most 5 mm. [3]

2.4 State the two checks that turn a claimed free response into an audited one. [4]

Exercises 2.3·2.4

2.3 The floor drives the rig at $r = 1$. The magnification, and the drive amplitude if the till may see at most 5 mm. [3]

2.3

- $M(1) = 1/(2 \times 0.2) = 2.5$.
- Drive $\leq 5/2.5 = 2$ mm.

2.4 State the two checks that turn a claimed free response into an audited one. [4]

2.4

- Substitute x, $\dot{x}$, $\ddot{x}$ from the claimed formula into the ODE: residual at machine precision.
- Check the initial conditions match, and the first-extremum value against $-e^{-\zeta\pi/\sqrt{1-\zeta^2}}$.