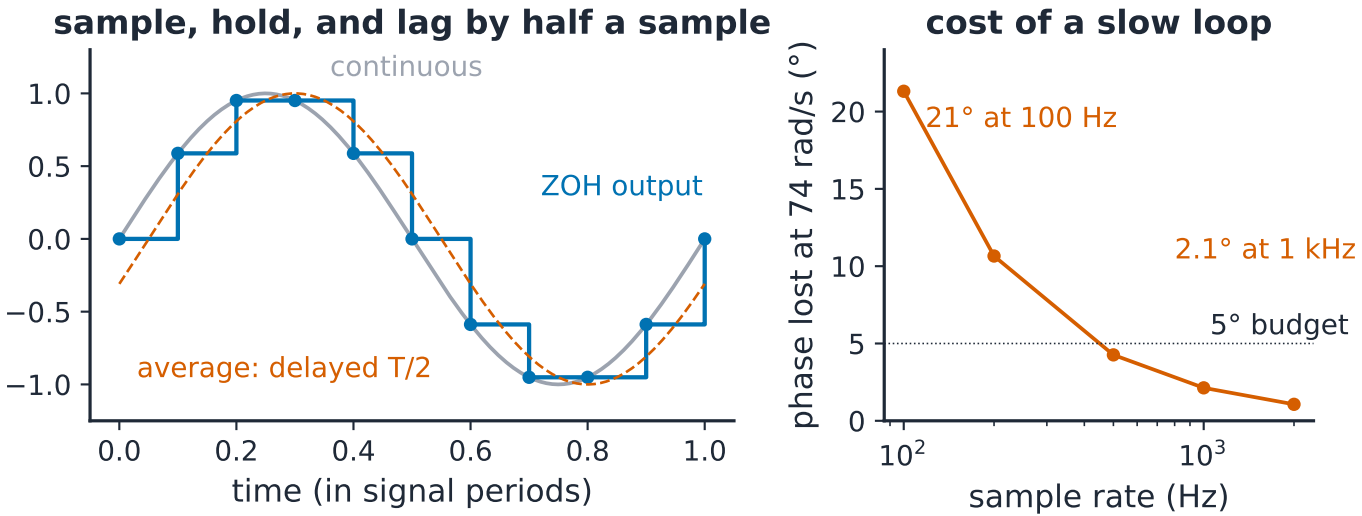
Exercise walk-through

# 12.5
# Digital implementation

University Mechanical Engineering

# Sampling costs phase




Hold lag $\omega T/2$ at 74.4 rad/s: $\mathbf{2.1}°$ at 1 kHz, $\mathbf{21}°$ at 100 Hz. Choose 1 kHz.

# Tustin, quantisation, latency

1. **Integral** $I_k = I_{k-1} + \frac{T}{2}(e_k + e_{k-1})$; $u_k = K_p e_k + K_i I_k$.
2. **Encoder** 2000 counts/rev $\times$ 50 $\rightarrow$ $\mathbf{0.0036}^\circ$ per count at the valve.
3. **Speed** 1 count per ms $= \mathbf{3.14}$ rad/s at the motor - too coarse for derivative action.
4. **Latency** 0.2 ms computation $\rightarrow$ 0.6° more lag at **crossover**.

## Decision

Position resolution is ample; speed estimates and delay are the real limits.

# Exercises 2.1·2.2

**2.1** Find the hold's phase lag at crossover (two decimals, degrees) and the remaining phase margin when sampling at 500 Hz and at 100 Hz. [4]

**2.2** Sampling at 1 kHz, where does noise at 1.2 kHz appear? And noise at 700 Hz? [2]

# Exercises 2.1·2.2

**2.1** Find the hold's phase lag at crossover (two decimals, degrees) and the remaining phase margin when sampling at 500 Hz and at 100 Hz. [4]

**2.1**

- Lag $= \omega_c T/2$: at 500 Hz, $29.8 \times 0.001$ rad $=$ **1.71**°; at 100 Hz, $29.8 \times 0.005$ rad $=$ **8.54**°.
- Margins: $PM' = PM - \omega_c T/2 = 65.2 - 1.7 =$ **63.5**° and $65.2 - 8.5 =$ **56.7**°.

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**2.2** Sampling at 1 kHz, where does noise at 1.2 kHz appear? And noise at 700 Hz? [2]

**2.2**

- $|1200 - 1000| =$ **200** Hz; $|700 - 1000| =$ **300** Hz: both fold below the 500 Hz Nyquist frequency, indistinguishable from real signals.

# Exercise 2.3

**2.3** A 1000-line encoder (quadrature $\times 4$) on the motor drives the valve through $30:1$. Find the counts per valve turn and the angle per count (degrees). [2]

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**2.3** A 1000-line encoder (quadrature $\times 4$) on the motor drives the valve through $30:1$. Find the counts per valve turn and the angle per count (degrees). [2]

**2.3**

- $1000 \times 4 \times 30 = \mathbf{120\,000}$ counts per valve turn.
- $360/120\,000 = \mathbf{0.003}^\circ$ per count.

# Exercise 3.1

**3.1** (a) Find the motor-speed resolution from counting that encoder at 1 kHz (three decimals, rad/s) and say what it implies for derivative action. (b) A 0.5 ms computation delay is added: find its lag at crossover (two decimals, degrees). (c) With $T = 2$ ms, $K_p = 79.4$, $K_i = 365.2$, $I_0 = 0$ and errors $e_0 = 0.10$, $e_1 = 0.08$, $e_2 = 0.06$ rad, find $I_1$, $I_2$ and $u_2$ (three decimals, V). [8]

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**3.1**

- (a) One count per sample: $2\pi/4000/0.001 = \mathbf{1.571}$ rad/s at the motor.
- The speed estimate jumps in steps of 1.57 rad/s, so $K_d\dot{e}$ would inject that noise: leave D out or filter heavily.
- (b) $\omega_c \tau_d = 29.8 \times 0.0005 = 0.0149$ rad $= \mathbf{0.85}^\circ$.
- (c) $I_1 = I_0 + \frac{T}{2}(e_1 + e_0) = 0.001 \times 0.18 = \mathbf{0.00018}$.
- $I_2 = I_1 + \frac{T}{2}(e_2 + e_1) = 0.00018 + 0.001 \times 0.14 = \mathbf{0.00032}$.
- $u_2 = K_p e_2 + K_i I_2 = 79.4 \times 0.06 + 365.2 \times 0.00032 = \mathbf{4.881}$ V.