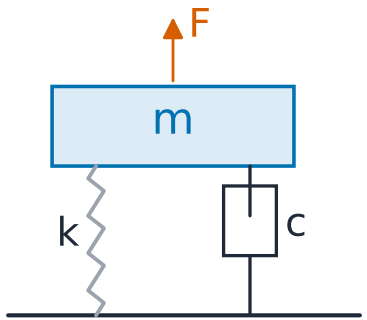
Exercise walk-through

12.1
Lumped-parameter modelling

University Mechanical Engineering

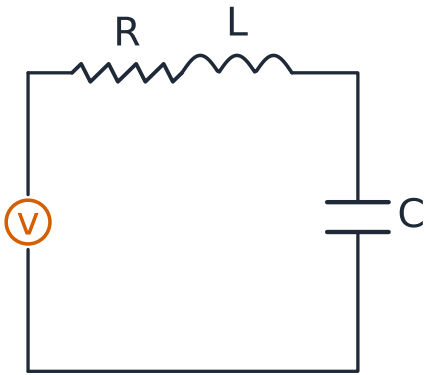
One equation form, three domains

mechanical



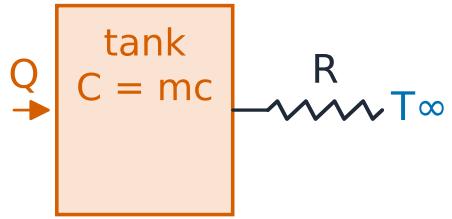
$$m\,\ddot{x} + c\,\dot{x} + k\,x = F$$

electrical



$$L\,\ddot{q} + R\,\dot{q} + q/C = v$$

thermal



$$C\,\dot{T} + (T - T_\infty)/R = Q$$

Reservoir 200 L: $C = \mathbf{836}$ kJ/K, $R = 0.01$ K/W, $\tau = RC = \mathbf{2.3}$ h; 1 kW raises it **10** K.

The mount in state space

1. **States** $x_1 = x,\ x_2 = \dot{x};\ m = 120,\ c = 910,\ k = 300\,000.$
2. **Matrices** $\mathbf{A} = [0, 1;\ -2500, -7.58],\ \mathbf{B} = [0;\ 1/120].$
3. **Eigenvalues** $-3.79 \pm 49.86j$ - the decay record's $\zeta\omega_n$ and ω_d.

Decision

One matrix form now serves the mount, the motor and the loop.

Exercises 2.1·2.2

2.1 A 150 L water tank ($c = 4180$ J/(kg · K)) loses heat through $UA = 50$ W/K. Find C (kJ/K), R (K/W) and τ (s and hours, two decimals). [3]

2.2 A 2 kW heater is switched on. Find the final temperature rise, the rise after 2 h (one decimal, K), and the time to reach a 30 K rise (two decimals, h). [4]

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2.1

- $C = mc = 150 \times 4180 = \mathbf{627}$ kJ/K.
- $R = 1/UA = \mathbf{0.02}$ K/W.
- $\tau = RC = 0.02 \times 627\,000 = \mathbf{12\,540}$ s $= \mathbf{3.48}$ h.

2.2 A 2 kW heater is switched on. Find the final temperature rise, the rise after 2 h (one decimal, K), and the time to reach a 30 K rise (two decimals, h). [4]

2.2

- Final rise $= QR = 2000 \times 0.02 = \mathbf{40}$ K.
- $\Delta T = QR(1 - e^{-t/\tau}) = 40(1 - e^{-7200/12\,540}) = \mathbf{17.5}$ K.
- $30 = 40(1 - e^{-t/\tau})$ gives $t = \tau \ln 4 = \mathbf{4.83}$ h.

Exercise 2.3

2.3 State one assumption that makes the tank a single lump. [1]

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2.3

- The water is well mixed, so it has one temperature (or: UA is constant; the wall stores no heat).

Exercise 3.1

3.1 (a) A 50 kg mass on a spring of 20 kN/m with a 400 N · s/m damper is pushed by a force F. Write **A** and **B**, find the eigenvalues of **A**, and from them ω_n and ζ. (b) A series RLC circuit has $L = 0.5$ H, $R = 40\ \Omega$ and $C = 500$ µF. Write its equation in the charge q, name the mechanical analogue of each element, and find ω_n (two decimals, rad/s) and ζ (three decimals). [9]

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3.1 (a) A 50 kg mass on a spring of 20 kN/m with a 400 N · s/m damper is pushed by a force F. Write $\mathbf{A}$ and $\mathbf{B}$, find the eigenvalues of $\mathbf{A}$, and from them ω_n and ζ. (b) A series RLC circuit has $L = 0.5$ H, $R = 40\ \Omega$ and $C = 500\ \mu$F. Write its equation in the charge q, name the mechanical analogue of each element, and find ω_n (two decimals, rad/s) and ζ (three decimals). [9]

3.1

- (a) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -400 & -8 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 \\ 0.02 \end{bmatrix}$.
- $\det(\lambda\mathbf{I} - \mathbf{A}) = \lambda^2 + 8\lambda + 400 = 0$, so $\lambda = \mathbf{-4 \pm 19.6j}$.
- $\omega_n = |\lambda| = \sqrt{400} = \mathbf{20}$ rad/s; $\zeta = 4/20 = \mathbf{0.2}$.
- (b) $L\ddot{q} + R\dot{q} + q/C = V$: inductance $\leftrightarrow$ mass, resistance $\leftrightarrow$ damper, $1/C \leftrightarrow$ spring stiffness, voltage $\leftrightarrow$ force.
- $\omega_n = 1/\sqrt{LC} = 1/\sqrt{0.5 \times 500 \times 10^{-6}} = \mathbf{63.25}$ rad/s.
- $\zeta = (R/2)\sqrt{C/L} = 20\sqrt{0.001} = \mathbf{0.632}$.