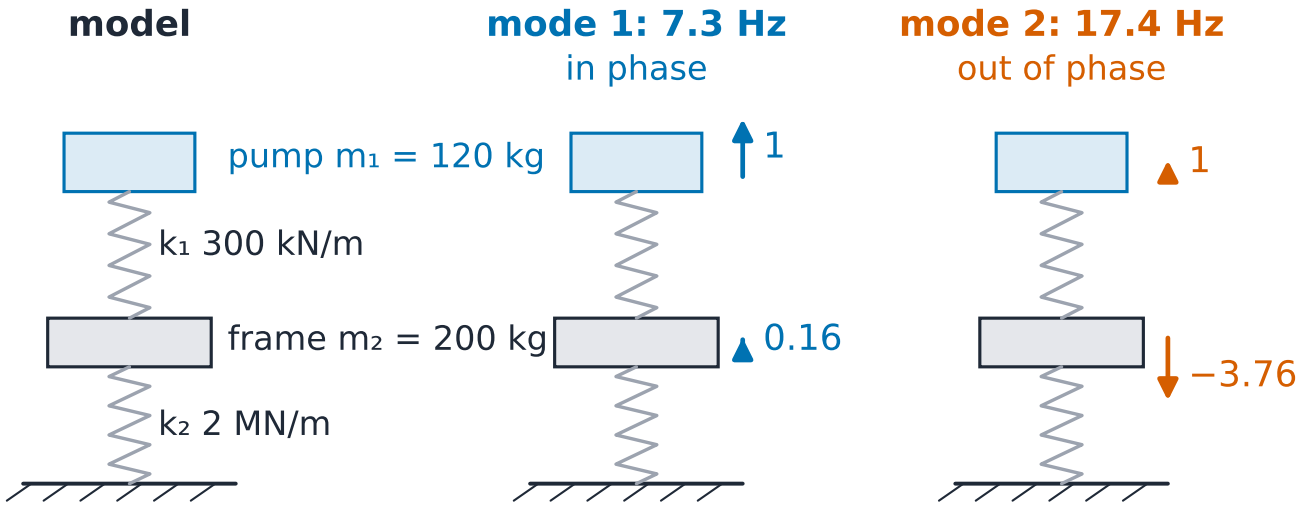
Exercise walk-through

11.6 Multi-degree-of-freedom systems

University Mechanical Engineering

Two masses, two modes



Pump 120 kg on 300 kN/m, frame 200 kg on 2 MN/m: **7.30** Hz in phase (1, 0.16), **17.4** Hz out of phase (1, −3.76).

Response at running speed

1. **Matrices** $\mathbf{M} = \mathrm{diag}(120, 200)$; $\mathbf{K} =$ [300, -300; -300, 2300] kN/m.
2. **Solve** $(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{X} = (27.3, 0)$ at 150.8 rad/s.
3. **Result** Pump $\mathbf{11.4}$ µm, frame $\mathbf{1.5}$ µm, floor $k_2 X_2 = \mathbf{3.0}$ N.

Decision

24 Hz lies above both modes, so the two-mass model confirms the single-mass estimate.

Check yourself

- The crank and the rod? (176.2 mm, 12.5°; 1.228 m/s, 4.02 rad/s, 14.8 m/s^2.)
- The shaft and the piston? (0.485 kg at 202°, 0.335 kg at 243°; 31.6 N and 7.9 N.)
- The mounts? (7.96 Hz, ζ 0.076; T 0.136 at 24 Hz; 6 Hz mounts give 0.078.)
- The torsion and the frame? (51.6 Hz; 7.30 and 17.4 Hz, floor force 3.0 N.)

Exercises 2.1·2.2

2.1 Write $\mathbf{M}$ and $\mathbf{K}$. [2]

2.2 Show that $\det(\mathbf{K} - \omega^2\mathbf{M}) = 0$ becomes $15\,000\,\omega^4 - 2.0 \times 10^8\,\omega^2 + 3.0 \times 10^{11} = 0$, and find both natural frequencies (two decimals, rad/s and Hz). [5]

Exercises 2.1·2.2

2.1 Write $\mathbf{M}$ and $\mathbf{K}$. [2]

2.1

- $\mathbf{M} = \mathrm{diag}(100, 150)$ kg.
- $\mathbf{K} = \begin{bmatrix} 200 & -200 \\ -200 & 1700 \end{bmatrix}$ kN/m.

2.2 Show that $\det(\mathbf{K} - \omega^2 \mathbf{M}) = 0$ becomes $15\,000\,\omega^4 - 2.0 \times 10^8\,\omega^2 + 3.0 \times 10^{11} = 0$, and find both natural frequencies (two decimals, rad/s and Hz). [5]

2.2

- $(k_1 - \omega^2 m_1)(k_1 + k_2 - \omega^2 m_2) - k_1^2 = 0$.
- Expanding: $m_1 m_2 \omega^4 - [m_1(k_1 + k_2) + m_2 k_1]\omega^2 + k_1 k_2 = 0$, that is $15\,000\omega^4 - 2.0 \times 10^8 \omega^2 + 3.0 \times 10^{11} = 0$.
- $\omega^2 = 1722.5$ or $11\,611$.
- $\omega_1 = \mathbf{41.50}$ rad/s (**6.61** Hz); $\omega_2 = \mathbf{107.75}$ rad/s (**17.15** Hz).

Exercise 3.1

3.1 (a) Find both mode shapes as $(1, u_2)$ (three decimals) and describe each. (b) The fan's unbalance gives $F_0 = 49.3$ N on m_1 at 157.1 rad/s. Find the pump and frame amplitudes (one decimal and two decimals, µm) from $(\mathbf{K} - \omega^2\mathbf{M})\mathbf{X} = (F_0, 0)$, and the floor force (one decimal, N). (c) Compare with treating the fan alone on k_1 above a rigid floor, and say what would happen if mode 2 moved to 25 Hz. [10]

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3.1

- (a) $u_2 = 1 - \omega^2 m_1/k_1 = 1 - \omega^2/2000$.
- Mode 1: $u_2 = 1 - \omega_1^2 m_1/k_1 = 1 - 1722.5/2000 = \mathbf{0.139}$: both move together, the mounts flex.
- Mode 2: $u_2 = 1 - \omega_2^2 m_1/k_1 = 1 - 11\,611/2000 = \mathbf{-4.805}$: the frame bounces against the fan on its stiff feet.
- (b) $\omega^2 = 24\,674$: $\mathbf{K} - \omega^2\mathbf{M} = \begin{bmatrix} -2.267 & -0.2 \\ -0.2 & -2.001 \end{bmatrix} \times 10^6$ N/m.
- Solving: $X_1 = \mathbf{-22.0}$ µm, $X_2 = \mathbf{2.19}$ µm; the fan moves in antiphase with the force (above both modes).
- Floor force $= k_2 X_2 = 1.5 \times 10^6 \times 2.19 \times 10^{-6} = \mathbf{3.3}$ N.
- (c) Alone on k_1: $X = F_0/(k_1 - \omega^2 m_1) = -21.8$ µm, and the floor would feel $k_1 X = 4.4$ N; the frame adds a little isolation.
- With mode 2 at 25 Hz, the running speed would excite it: the frame amplitude and floor force would rise sharply, so every mode must be checked, not only the first.