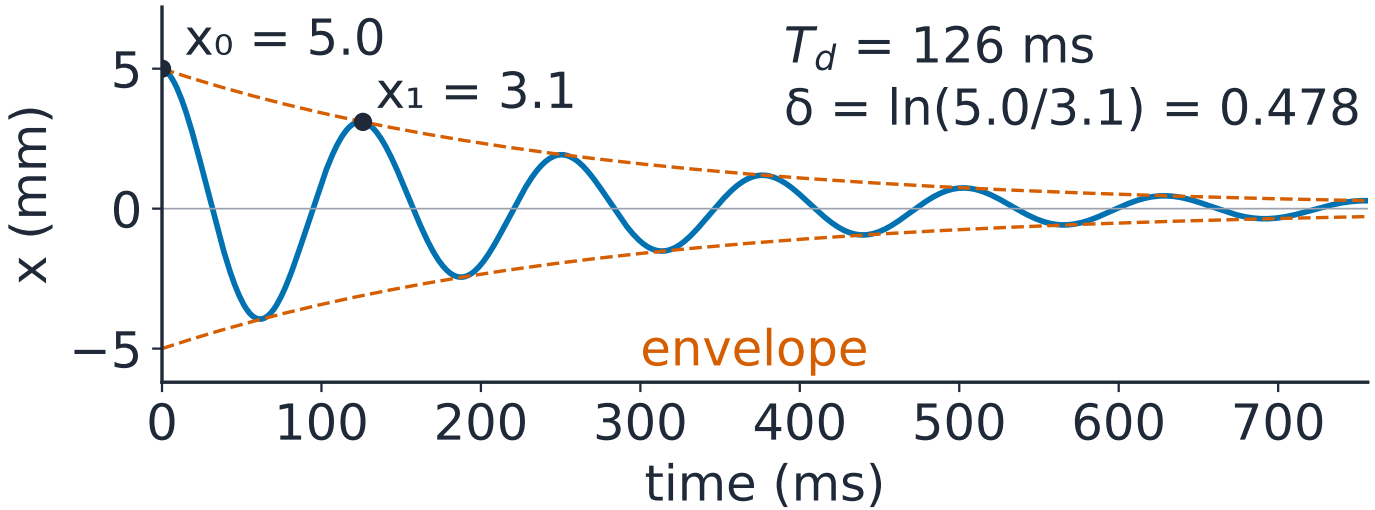
Exercise walk-through

11.4
Free vibration

University Mechanical Engineering

Free decay and damping



$k = 300$ kN/m, $m = 120$ kg: $f_n = \mathbf{7.96}$ Hz. Peaks 5.0 → 3.1 mm: $\delta = 0.478$, $\zeta = \mathbf{0.076}$.

Torsion between gear and impeller

1. **Stiffness** Ø30, 0.6 m, $G = 80$ GPa $\rightarrow$ $k_t = GJ_p/L = \mathbf{10.6}$ kN $\cdot$ m/rad.
2. **Inertias** Gear $J_1 = 0.64$, impeller $J_2 = 0.12$ kg $\cdot$ m^2.
3. **Frequency** $\omega = \sqrt{k_t(J_1 + J_2)/J_1 J_2} = \mathbf{324}$ rad/s $= 51.6$ Hz, node 95 mm from the gear.

Decision

3090 rpm against 180 rpm running - no torsional **resonance**.

Exercises 2.1·2.2

2.1 A 200 kg compressor stands on four mounts of 100 kN/m each. Find ω_n (two decimals, rad/s), f_n (two decimals, Hz) and the static deflection (one decimal, mm). [4]

2.2 A tap test gives successive peaks of 4.0 mm and 2.8 mm one period apart. Find δ (four decimals), ζ (four decimals), c (nearest N · s/m) and ω_d (two decimals, rad/s). [5]

Exercises 2.1·2.2

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2.1

- $k = 4k_{mount} = 4 \times 100 = 400$ kN/m; $\omega_n = \sqrt{k/m} = \sqrt{400\,000/200} = \mathbf{44.72}$ rad/s.
- $f_n = \omega_n / 2\pi = \mathbf{7.12}$ Hz.
- $\delta_{st} = mg/k = 200 \times 9.81/400\,000 = \mathbf{4.9}$ mm.

2.2 A tap test gives successive peaks of 4.0 mm and 2.8 mm one period apart. Find δ (four decimals), ζ (four decimals), c (nearest N · s/m) and ω_d (two decimals, rad/s). [5]

2.2

- $\delta = \ln(x_0/x_1) = \ln(4.0/2.8) = \mathbf{0.3567}$.
- $\zeta = \delta / \sqrt{4\pi^2 + \delta^2} = 0.3567/6.293 = \mathbf{0.0567}$.
- $c = 2\zeta\sqrt{km} = 2 \times 0.0567 \times \sqrt{400\,000 \times 200} = \mathbf{1014}$ N · s/m.
- $\omega_d = \omega_n\sqrt{1-\zeta^2} = \mathbf{44.65}$ rad/s, almost unchanged.

Exercise 3.1

3.1 A steel shaft ($G = 80$ GPa), 40 mm in diameter and 0.8 m long, joins a flywheel ($J_1 = 1.0\ \text{kg}\cdot\text{m}^2$) to a fan ($J_2 = 0.25\ \text{kg}\cdot\text{m}^2$). (a) Find J_p and k_t (one decimal, kN · m/rad). (b) Find the torsional natural frequency in rad/s, Hz and rpm. (c) Locate the node measured from the flywheel, and say whether running at 1500 rpm is safe. [8]

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3.1

- (a) $J_p = \pi d^4/32 = \pi \times 0.04^4/32 = 2.513 \times 10^{-7}$ m^4.
- $k_t = GJ_p/L = 80 \times 10^9 \times 2.513 \times 10^{-7}/0.8 = \mathbf{25.1}$ kN $\cdot$ m/rad.
- (b) $\omega = \sqrt{k_t(J_1 + J_2)/(J_1 J_2)} = \sqrt{25\,133 \times 1.25/0.25} = \mathbf{354.5}$ rad/s.
- $f = 354.5/2\pi = \mathbf{56.4}$ Hz $= \mathbf{3385}$ rpm.
- (c) The discs swing in opposite directions with amplitudes in the ratio $J_2 : J_1$, so the node is $LJ_2/(J_1 + J_2) = 0.8 \times 0.25/1.25 = \mathbf{0.16}$ m from the flywheel.
- 1500 rpm is 44% of 3385 rpm, well clear of resonance; a 2nd-order excitation (3000 rpm) would come close.