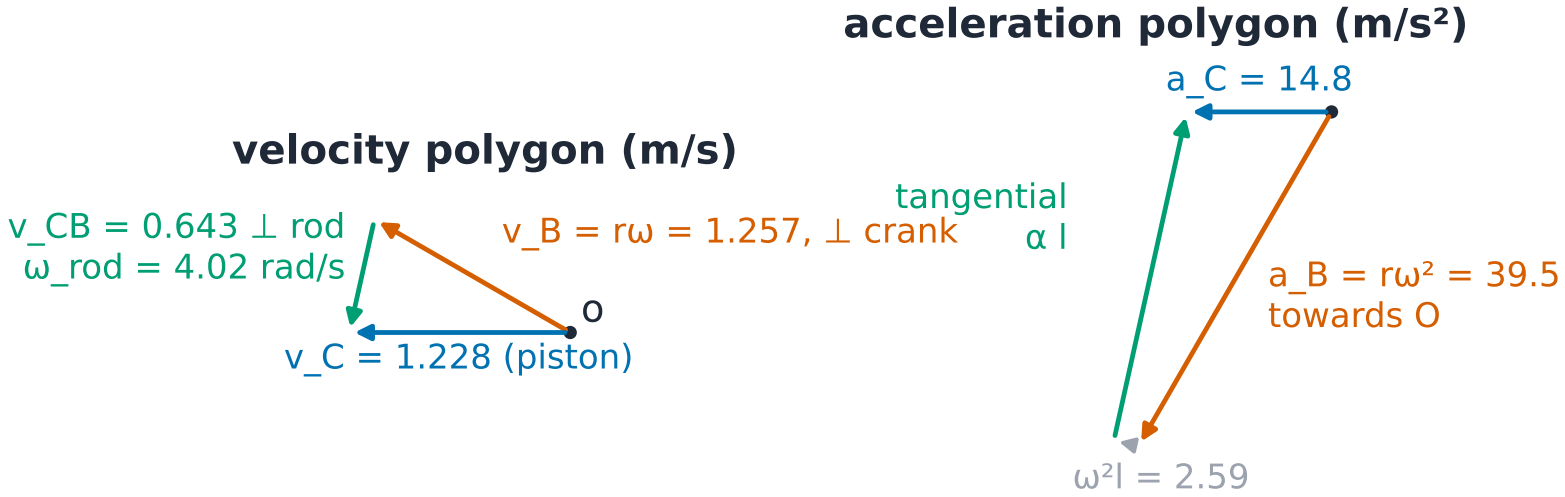
Exercise walk-through

11.2 Velocity and acceleration analysis

University Mechanical Engineering

Velocity and acceleration polygons



300 rpm, 60°: $v_B = \mathbf{1.257}$, $v_C = \mathbf{1.228}$ m/s, $\omega_{rod} = \mathbf{4.02}$ rad/s; $a_B = 39.5$, $a_C = \mathbf{14.8}$ m/s^2.

Two checks and Coriolis

1. **Instant centre** Rod about I: $1.257/0.312 = 1.228/0.305 = \mathbf{4.02}$ rad/s.
2. **Formula** $a_C \approx r\omega^2(\cos\theta + \cos 2\theta/n) = 14.80$ m/s^2, $n = 4$.
3. **Coriolis** Block at 0.6 m/s on an arm at 5 rad/s: $2\omega v_{rel} = \mathbf{6.0}$ m/s^2.

Decision

Two methods that agree are worth more than one careful one.

Exercises 2.1·2.2

2.1 Find ω (two decimals, rad/s) and the crank-pin speed v_B (three decimals, m/s). [2]

2.2 At $\theta = 45^\circ$ ($\phi = 10.18^\circ$) find the piston speed v_C (three decimals, m/s), the rod's angular velocity ω_3 (two decimals, rad/s) and v_{CB}. [5]

Exercises 2.1·2.2

2.1 Find ω (two decimals, rad/s) and the crank-pin speed v_B (three decimals, m/s). [2]

2.1

- $\omega = 2\pi N/60 = 2\pi \times 600/60 = \mathbf{62.83}$ rad/s.
- $v_B = r\omega = 0.05 \times 62.83 = \mathbf{3.142}$ m/s.

2.2 At $\theta = 45°$ ($\phi = 10.18°$) find the piston speed v_C (three decimals, m/s), the rod's angular velocity ω_3 (two decimals, rad/s) and v_{CB}. [5]

2.2

- $v_C = r\omega \sin(\theta + \phi)/\cos\phi = 3.142 \sin 55.18°/\cos 10.18° = \mathbf{2.620}$ m/s.
- $\omega_3 = r\omega \cos\theta/(l\cos\phi) = 3.142 \cos 45°/(0.2 \cos 10.18°) = \mathbf{11.28}$ rad/s.
- $v_{CB} = \omega_3 l = 11.28 \times 0.2 = \mathbf{2.257}$ m/s, across the rod.

Exercise 2.3

2.3 A block slides outwards at 0.4 m/s along a slotted arm turning at 8 rad/s. Find the Coriolis acceleration and its direction. [3]

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2.3

- $a_{Cor} = 2\omega v_{rel} = 2 \times 8 \times 0.4 = \mathbf{6.4}$ m/s^2.
- Perpendicular to the arm, towards the side the arm is turning (the sliding velocity rotated 90° with ω).

Exercise 3.1

3.1 For the slider-crank of 2.2 at $\theta = 45°$: (a) find the crank-pin acceleration and the rod's centripetal acceleration (one decimal, $\mathrm{m/s^2}$); (b) estimate the piston acceleration with the approximate formula (one decimal, $\mathrm{m/s^2}$) and give its direction; (c) find the inertia force on a 1.2 kg piston (nearest N) and say why the approximation is good here. [7]

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3.1

- (a) $a_B = r\omega^2 = 0.05 \times 62.83^2 = \mathbf{197.4}$ m/s^2, towards O.
- Rod centripetal $\omega_3^2 l = 11.28^2 \times 0.2 = \mathbf{25.5}$ m/s^2, from C towards B.
- (b) $n = l/r = 4$; $a_C \approx r\omega^2(\cos\theta + \cos 2\theta/n) = 197.4(0.7071 + 0) = \mathbf{139.6}$ m/s^2.
- Directed towards the crank: past outer dead centre the piston is speeding up as it moves in.
- (c) $F = ma_C = 1.2 \times 139.6 = \mathbf{168}$ N.
- The dropped terms are of order $(r/l)^3$; with $n = 4$ the exact polygon gives 140.4 m/s^2, 0.6% more.