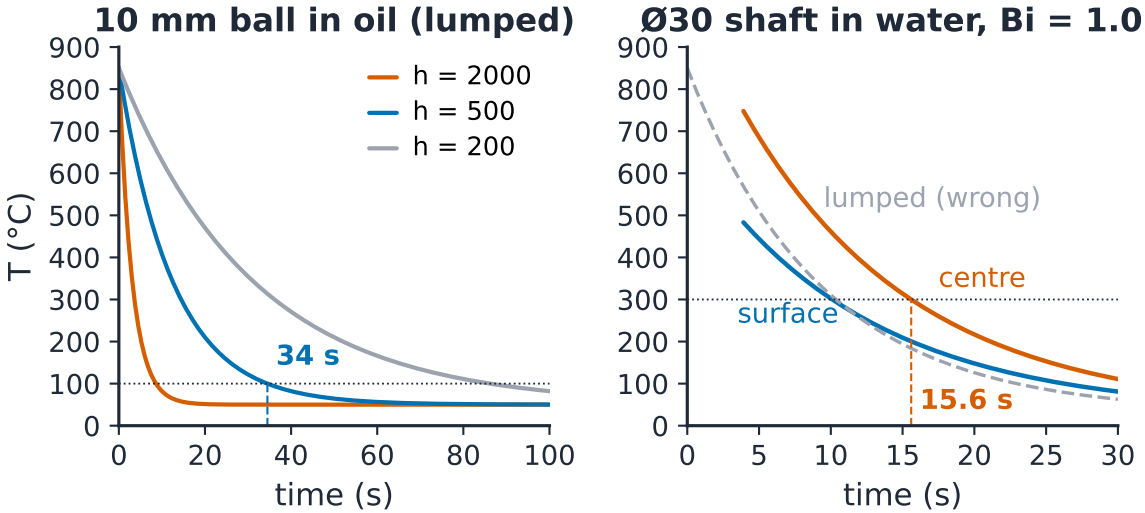
Exercise walk-through

10.2
Transient conduction

University Mechanical Engineering

Quenching curves



Ball: $Bi = 0.019$, $\tau = \mathbf{12.4}$ s, 850 → 100 °C in $\mathbf{34}$ s. Shaft: $Bi = 1.05$, centre to 300 °C in $\mathbf{15.6}$ s.

When lumped is wrong

1. **Biot** $Bi = 3000 \times 0.015/43 = \mathbf{1.05}$ - not lumped.
2. **One term** $\zeta_1 = 1.278$, $C_1 = 1.215$; $\theta_0 = 270/820 = 0.329$.
3. **Time** $Fo = \ln(C_1/\theta_0)/\zeta_1^2 = \mathbf{0.80}$, $t = \mathbf{15.6}$ s; surface about 200 °C.
4. **Lumped** Would say 10.4 s - 33% short, and blind to the 100 K gradient.

Decision

Above $Bi = 0.1$ the inside matters; the lumped answer is too optimistic.

Exercises 2.1·2.3

2.1 A 20 mm steel ball is quenched in oil at 40 °C with $h = 300$ W/(m^2 · K). Find L_c (two decimals, mm) and Bi (four decimals). Is lumped valid? [3]

2.2 Find the time constant (one decimal, s) and the time to cool from 800 to 100 °C (nearest s). [4]

2.3 Find the ball's temperature 60 s after it enters the oil (nearest °C). [3]

Exercises 2.1·2.3

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2.1

- $L_c = r/3 = 10/3 = \mathbf{3.33}$ mm.
- $Bi = hL_c/k = 300 \times 0.00333/43 = \mathbf{0.0233}$, below 0.1: lumped is valid.

2.2 Find the time constant (one decimal, s) and the time to cool from 800 to 100 °C (nearest s). [4]

2.2

- $\tau = \rho c L_c/h = 7850 \times 475 \times 0.00333/300 = \mathbf{41.4}$ s.
- $t = \tau \ln(\theta_i/\theta) = 41.4 \ln(760/60) = \mathbf{105}$ s.

2.3 Find the ball's temperature 60 s after it enters the oil (nearest °C). [3]

2.3

- $T = T_\infty + \theta_i e^{-t/\tau} = 40 + 760e^{-60/41.4} = \mathbf{219}$ °C.

Exercise 3.1

3.1 A 40 mm steel shaft at 850 °C is quenched in water at 30 °C with $h = 2000$ W/(m^2 · K). Find $Bi = hr_o/k$ and say why lumped fails. For this Bi, $\zeta_1 = 1.221$ and $C_1 = 1.195$. Find the time for the centre to reach 300 °C (one decimal, s), and compare with the lumped estimate (one decimal, s). [7]

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3.1

- $Bi = hr_o/k = 2000 \times 0.020/43 = \mathbf{0.93}$, far above 0.1: the centre lags the surface.
- $\theta_0/\theta_i = 270/820 = 0.329$; $Fo = \ln(C_1/0.329)/\zeta_1^2 = \ln(1.195/0.329)/1.221^2 = 0.865$.
- $\alpha = k/(\rho c) = 1.153 \times 10^{-5}$ m^2/s; $t = Fo\, r_o^2/\alpha = 0.865 \times 0.02^2/(1.153 \times 10^{-5}) = \mathbf{30.0}$ s.
- Lumped: $\tau = \rho c(r/2)/h = 18.6$ s and $t = 18.6 \ln(820/270) = \mathbf{20.7}$ s, a third too short.