Topic 09

University Mechanical Engineering

What this course is, and how to use it

This is the ninth course in University Mechanical Engineering: **machine design** – turning loads into parts that survive them. We design the drive of the cooling-loop pump: a motor at 1440 rpm, a spur-gear stage down to 180 rpm, the 300 N · m output shaft from Course 6, its bearings, and the bolts that hold the housing down.

Scope limit, stated once: a study course, not a degree; there is no design office. Material data are typical handbook values for C45 (AISI 1045, cold drawn: $S_{ut} = 630$ MPa, $S_y = 530$ MPa), bearing and bolt data come from standard catalogue tables, and every number printed here is recomputed in the course suite.

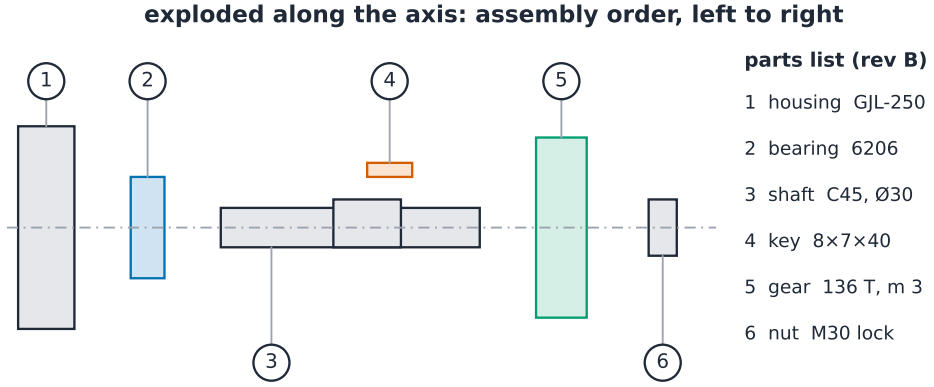
- Unit 9.1 says how a design is written down and kept under control.
- Units 9.2–9.3 set the rules: load paths, safety factors, static and fatigue criteria.
- Units 9.4–9.5 apply them to shafts, bearings, bolts and gears.
- Unit 9.6 runs the whole drive as one constrained design project.

The habit: **every part gets a load, a criterion and a safety factor in writing** – a design without its numbers is a drawing, not a design.

Design communication and CAD

Why it matters. The drive has six parts made by three suppliers. If the gear maker works from revision A and the shaft maker from revision B, the key will not fit. Design communication is how a team avoids that.

The idea, with a picture. An **assembly drawing** shows how the parts go together; an **exploded view** pulls them apart along the assembly direction, and **balloons** link each part to the **bill of materials** (BOM). In CAD, **parametric modelling** stores sizes as named parameters linked by relations, so a change propagates; **design intent** is the choice of which sizes drive and which follow. **Revision control** gives every released drawing a letter and records each change.



An exploded view shows assembly order along the axis; balloons tie each part to the parts list

The derivation (why intent decides robustness). Suppose the key length is drawn as a fixed 40 mm while the gear hub length is a parameter. A later hub change from 45 to 36 mm leaves a key longer than its hub – the model rebuilds without error, but the part is wrong. If instead the key length is related to the hub ($L_{key} = L_{hub} - 5$), the same change gives a 31 mm key. Encoding the relation is encoding the reason.

> **Worked example 1 – choosing the driving dimensions.** The shaft seats are driven by the bearing bore (30 mm, from the 6206 catalogue entry); the shoulder is driven by the bearing's recommended abutment diameter (36 mm); the gear seat follows the seat diameter. Change the bearing to a 6207 (35 mm bore) and the seat, shoulder (42 mm) and gear bore all update from one parameter.

> **Worked example 2 – a controlled change.** The gear supplier asks to widen the face from 30 to 32 mm. The change note records: reason (supplier tooling), parts affected (gear 5, key 4, spacer), checks repeated (gear bending stress falls by 6%, shaft span unchanged) and the new revision letter C on drawings 4 and 5. Nobody needs to guess which drawing is current.

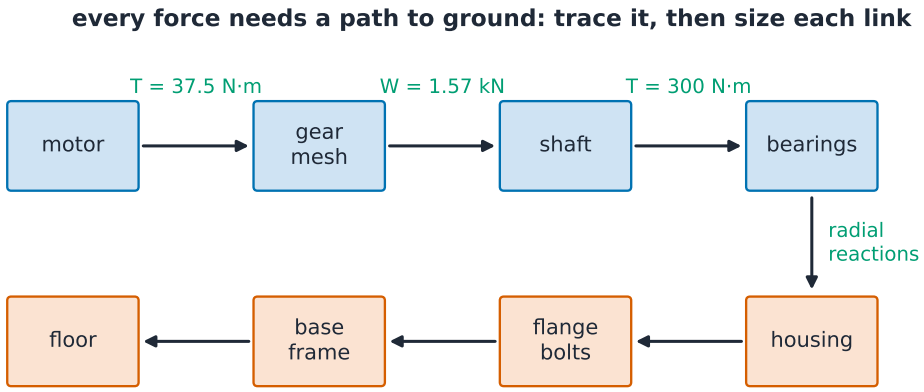
Common mistakes. Dimensioning to make the CAD easy rather than to show function. Releasing drawings without revision letters. Leaving parts out of the BOM because they are "standard" (bolts, keys, seals).

Summary. Exploded view + balloons + BOM; parameters with relations encode design intent; every release has a revision and a change record.

Load paths and design safety

Why it matters. Most machine failures happen at a link nobody checked: a key, a weld, a bolt. Tracing the **load path** from the source to the ground lists every link that needs a calculation.

The idea, with a picture. Follow the force: motor torque → gear mesh → shaft → bearings → housing → flange bolts → base frame → floor. Each arrow is an interface, and each interface is a possible failure. Then choose a **factor of safety** n = strength/stress that covers what you do not know.



The load path of the pump drive, from motor torque to the floor

The derivation (building a safety factor from its sources). If the load may be 20% higher than estimated ($n_L = 1.2$), the material 15% weaker than the handbook ($n_S = 1.15$) and the model 10% optimistic ($n_M = 1.1$), the factors multiply: $n = 1.2 \times 1.15 \times 1.1 = \mathbf{1.52}$. Each factor names an uncertainty; a single "use 2" hides them. Add margin where failure is dangerous.

> **Worked example 1 – converting a duty cycle.** The pump's side load on the bearings varies: 2 kN for 60% of the time, 3 kN for 30% and 5 kN for 10%. Ball-bearing life goes as $1/P^3$, so the equivalent load is the cube mean: $P_{eq} = (0.6 \times 2^3 + 0.3 \times 3^3 + 0.1 \times 5^3)^{1/3} = \mathbf{2.94}$ kN – well above the plain average of 2.6 kN, because the rare 5 kN peaks do most of the damage.

> **Worked example 2 – forces along the path.** The motor gives $T = 300/8 = \mathbf{37.5}$ N · m at the pinion. With pitch diameter 51 mm, the tooth force is $W_t = 2T/d = \mathbf{1471}$ N tangential, $W_r = W_t \tan 20° = \mathbf{535}$ N radial, $W = \mathbf{1565}$ N total. The output shaft then carries 300 N · m to the pump and bends under W; the bearings react W (1.02 kN and 0.55 kN for a gear 70 mm from one bearing on a 200 mm span) plus the hydraulic side load.

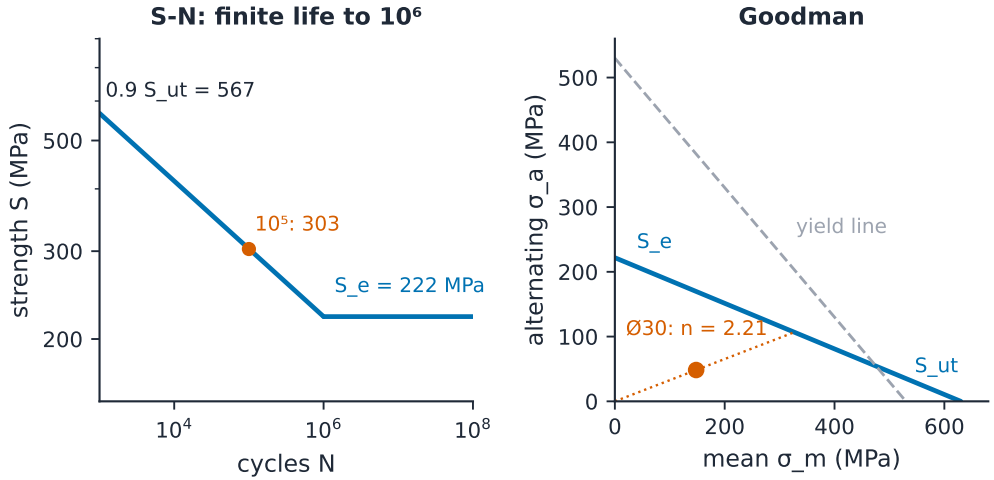
Common mistakes. Checking the shaft and forgetting the key and bolts. Using the average load where the life law is non-linear. Choosing a safety factor before listing the uncertainties.

Summary. Trace the path, size every link; $n = n_L n_S n_M$ from named uncertainties; duty cycles become cube-mean loads for bearings.

Static and fatigue criteria

Why it matters. The shaft turns, so each point on its surface sees bending stress swing from tension to compression once per turn – about 10^8 times in its life. Parts like this fail by **fatigue** at stresses well below yield.

The idea, with a picture. Static loads use the Course 6 criteria: **distortion energy** (von Mises) for ductile parts, **Coulomb–Mohr** – or maximum normal stress for brittle ones. Fatigue uses an **S-N curve** S-N : strength falls with cycles until the **endurance limit** S_e for steels. The **Goodman line** combines alternating and mean stress: $\sigma_a/S_e + \sigma_m/S_{ut} = 1/n$.



S-N line with its endurance limit, and the Goodman diagram for the shaft shoulder

The derivation (the endurance limit of a real part). A polished test bar of steel lasts forever below $S'_e \approx 0.5 S_{ut}$. A real part is worse, so **Marin factors** reduce it: $S_e = k_a k_b k_c \ldots S'_e$, with surface $k_a = a S_{ut}^b$ (machined: $a = 4.51$, $b = -0.265$) and size $k_b = 1.24 d^{-0.107}$ ($2.79 \le d \le 51$ mm). A notch raises local stress by the fatigue factor $K_f = 1 + q(K_t - 1)$, where q is the notch sensitivity.

Worked example 1 – the shaft's endurance limit. C45, machined, $d = 30$ mm: $S'_e = 0.5 \times 630 = 315$ MPa; $k_a = 4.51 \times 630^{-0.265} = \mathbf{0.817}$; $k_b = 1.24 \times 30^{-0.107} = \mathbf{0.862}$; $S_e = 0.817 \times 0.862 \times 315 = \mathbf{222}$ MPa. The S-N line from $0.9S_{ut} = 567$ MPa at 10^3 cycles to S_e at 10^6 gives $S = \mathbf{303}$ MPa at 10^5 cycles – the strength for a part that only needs a short life.

Worked example 2 – Goodman at the shoulder. At the gear-seat shoulder ($K_t = 2.0$, $q = 0.8$, so $K_f = \mathbf{1.8}$; torsion $K_{fs} = \mathbf{1.51}$), the 30 mm shaft carries an alternating bending moment $M_a = 71.2$ N · m and a steady torque $T_m = 300$ N · m: $\sigma_a = K_f \cdot 32M_a/\pi d^3 = \mathbf{48.4}$ MPa; $\tau_m = K_{fs} \cdot 16T_m/\pi d^3 = 85.4$ MPa, so the von Mises mean $\sigma_m = \sqrt{3}\,\tau_m = \mathbf{148.0}$ MPa. Goodman: $1/n = 48.4/222 + 148.0/630$, so $n = \mathbf{2.21}$. First-cycle yield: $n_y = 530/\sqrt{48.4^2 + 148.0^2} = \mathbf{3.40}$. Fatigue governs.

Common mistakes. Using $S'_e = 0.5S_{ut}$ without Marin factors. Applying K_t instead of K_f in fatigue. Forgetting that a steady torque still adds a mean stress.

Summary. Von Mises for ductile, Coulomb–Mohr for brittle; $S_e = k_a k_b \ldots 0.5S_{ut}$; $K_f = 1 + q(K_t - 1)$; Goodman $\sigma_a/S_e + \sigma_m/S_{ut} = 1/n$.

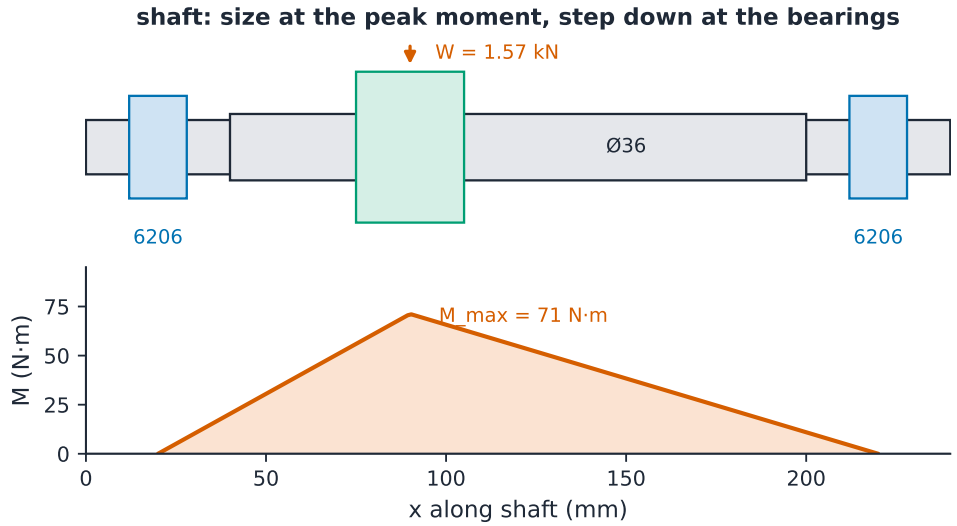
Shafts and bearings

Why it matters. The shaft must be strong enough and stiff enough, and the bearings must last as long as the pump. Both are chosen from the same loads.

The idea, with a picture. Draw the shaft's bending-moment diagram from the gear force and the bearing reactions; size the diameter at the worst point (here the gear seat, where moment and stress concentration meet); step the shaft down where the moment is small. For a rotating shaft with steady torque, the **DE-Goodman** shaft equation solves Goodman for d:

$$d = \left[\frac{16n}{\pi}\left(\frac{2K_f M_a}{S_e} + \frac{\sqrt{3}\,K_{fs}T_m}{S_{ut}}\right)\right]^{1/3}$$

For **rolling bearings**, the catalogue gives the **basic dynamic load rating** C; the **L10 life** (revolutions that 90% survive) is $L_{10} = (C/P)^3 \times 10^6$ for ball bearings.



The gear shaft between two bearings and its bending-moment diagram

The derivation (why the cube). Rolling-contact fatigue life falls with contact stress to a high power; for point contact (balls), tests show life $\propto P^{-3}$. So doubling the load cuts life to an eighth, and a required life fixes $C = P(L/10^6)^{1/3}$.

Worked example 1 – sizing the seat. With the shoulder data above and $n = 2$: $d = [\frac{32}{\pi}(2 \times 1.8 \times 71.2/221.8 + \sqrt{3} \times 1.51 \times 300/630)]^{1/3} = \mathbf{29.0}$ mm – choose the standard **30** mm, which gives $n = 2.21$ as found in unit 9.3. Peak moment $M = R_1 \times 70$ mm $= 1.017 \times 70 = \mathbf{71.2}$ N · m.

Worked example 2 – choosing the bearing. Required life 20 000 h at 180 rpm: $L = 180 \times 60 \times 20\,000 = \mathbf{2.16 \times 10^8}$ rev. With the duty-cycle load $P = 2.94$ kN: $C_{req} = 2.94 \times (216)^{1/3} = \mathbf{17.6}$ kN. The 6206 (30 mm bore, $C = 19.5$ kN) gives $L_{10} = (19.5/2.94)^3 \times 10^6/(60 \times 180) = \mathbf{27\,000}$ h. It fits the 30 mm seat and meets the life with 35% margin.

Common mistakes. Sizing at the peak moment but ignoring the notch next to it. Choosing a bearing by bore alone. Forgetting shaft deflection limits at the gear mesh (slope at the gear should stay below about 0.0005 rad).

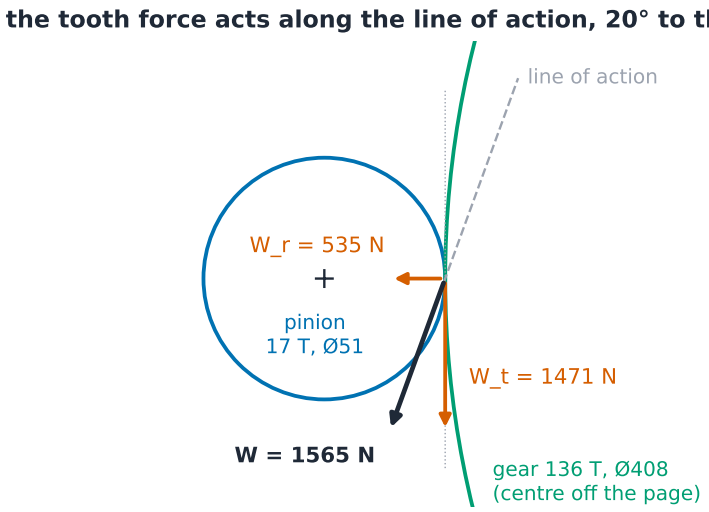
Summary. Moment diagram → worst section → DE-Goodman d; ball bearings $L_{10} = (C/P)^3 \times 10^6$ rev; $C_{req} = P(L/10^6)^{1/3}$.

Joints and gears

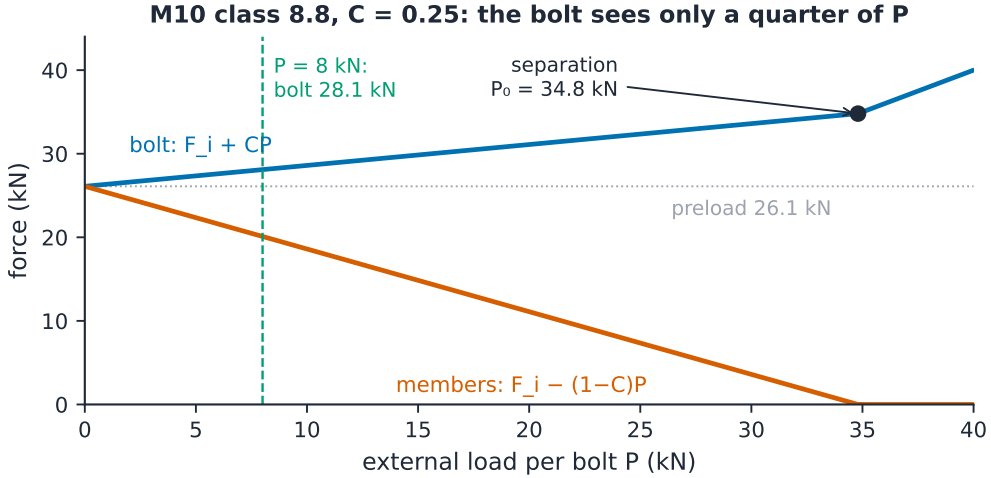
Why it matters. The housing flange bolts and the gear teeth are the two most loaded small features in the drive. Both have simple, reliable checks.

The idea, with a picture. A **bolted joint** is tightened to a **preload**

F_i (typically $0.75 A_t S_p$ for a reusable joint). Under an external load P, the bolt and the clamped members share it by stiffness: the bolt gets CP with $C = k_b/(k_b + k_m)$, and the joint separates at $P_0 = F_i/(1 - C)$. In a **spur gear** pair, the force acts along the **line of action** at the **pressure angle** $\phi = 20°$; the **module** $m = d/N$ sets the tooth size.



Spur-gear mesh: the tooth force acts along the line of action and splits into tangential and radial parts



Joint diagram: the preloaded bolt carries only its stiffness share of the external load until the joint separates

The derivation (the Lewis bending equation). Treat one tooth as a cantilever loaded at its tip by W_t. The stress at its root is $\sigma = W_t/(F\,m\,Y)$, where F is the face width and the **Lewis form factor** Y (0.303 for 17 teeth) carries the tooth shape. A dynamic factor $K_v = (6.1 + V)/6.1$ (cut teeth, V in m/s) adds the effect of speed: $\sigma = K_v W_t/(FmY)$.

> **Worked example 1 – the flange bolts.** M10 class 8.8: $A_t = 58.0$ mm^2, proof strength $S_p = 600$ MPa. Preload $F_i = 0.75 \times 58.0 \times 600 =$ **26.1** kN. With $C = 0.25$ and $P = 8$ kN per bolt: bolt force $= 26.1 + 0.25 \times 8 =$ **28.1** kN, stress **484** MPa, below the proof strength; load factor $n = (S_p A_t - F_i)/(CP) =$ **4.35**; the joint stays closed up to $P_0 = 26.1/0.75 =$ **34.8** kN.

> **Worked example 2 – the pinion teeth.** Module 3, 17 teeth ($d = 51$ mm), mating with 136 teeth ($d = 408$ mm, ratio 8:1); face $F = 30$ mm. At 1440 rpm the pitch-line speed is $V =$ **3.85** m/s, so $K_v =$ **1.63**. Bending: $\sigma = 1.63 \times 1471/(30 \times 3 \times 0.303) =$ **87.9** MPa. Contact (steel, $C_p = 191\sqrt{\text{MPa}}$, curvature radii 8.72 and 69.8 mm):

$$\sigma_c = 191\sqrt{\frac{1.63 \times 1471}{30\cos 20°}\left(\frac{1}{8.72} + \frac{1}{69.8}\right)} = \mathbf{633}\ \text{MPa}$$

The contact stress, not bending, will set the gear's material and hardness.

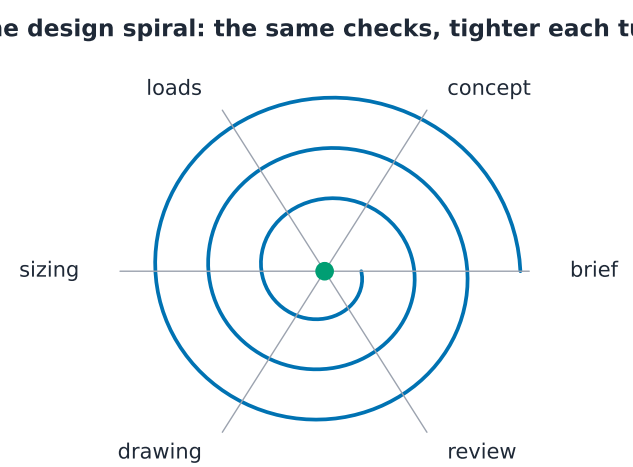
Common mistakes. Ignoring preload and adding the whole P to the bolt. Confusing module with diametral pitch. Checking bending but not surface contact on hardened gears.

Summary. $F_i = 0.75 A_t S_p$; bolt $F_i + CP$; separation $F_i/(1 - C)$; $W_t = 2T/d$, $W_r = W_t \tan\phi$; Lewis $\sigma = K_v W_t/(FmY)$; contact stress usually governs.

Constrained design project

Why it matters. Real design has limits on space, cost and time, and several answers that work. The project is to deliver one checked drive and say honestly where it stops being valid.

The idea, with a picture. Design moves round a **design spiral** : brief → concept → loads → sizing → drawing → review, then round again with better numbers. Concepts are compared with a **weighted decision matrix** , and the result is a design report that lists every check with its safety factor and its limits.



The design spiral: each turn repeats the same checks with better numbers

The derivation (why weights come first). If the weights are chosen after the scores, it is easy (even without meaning to) to pick weights that favour the concept you already like. Fixing the weights from the brief first – cost, size, efficiency, maintenance – makes the matrix a test, not a decoration.

> **Worked example 1 – choosing the drive concept.** Weights from the brief: cost 0.35, size 0.25, efficiency 0.25, maintenance 0.15. Scores (1–5): A spur gears (4, 3, 4, 3) → **3.60**; B belt drive (5, 2, 3, 3) → **3.45**; C gearmotor unit (3, 4, 4, 5) → **3.80**. C wins on this brief – but the course designs A, because it teaches the checks; the report must say so.

> **Worked example 2 – the design file.** One page lists the drive's checks: shaft Goodman $n = 2.21$, yield $n = 3.40$; bearing 6206 $L_{10} = 27\,000$ h against 20 000 h; bolts M10 8.8 load factor 4.35, separation at 34.8 kN; pinion bending 87.9 MPa, contact 633 MPa (needs through-hardened steel, about 300 HB or more). Limits stated: no shock loads, no shaft-deflection or key check yet, lubrication assumed adequate – the next turn of the spiral.

Common mistakes. Scoring before weighting. Reporting only the checks that passed. Calling a design finished when its limits are not written down.

Summary. Spiral through the same checks; weights before scores; the report lists every check, every factor and every limit.