Topic 07

University Mechanical Engineering

What this course is, and how to use it

This is the seventh course in University Mechanical Engineering: **fluid mechanics** – how liquids and gases push on surfaces, carry momentum and lose energy, worked on the cooling-water loop that the Course 6 drive shaft turns: one pump, one 80 mm pipe, one tank.

Scope limit, stated once: a study course, not a degree; the fluids are water at 20 °C ($\rho = 998$ kg/m^3, $\mu = 1.002 \times 10^{-3}$ Pa s) and air ($\rho = 1.204$ kg/m^3), $g = 9.81$ m/s^2, and every number printed here is recomputed in the course suite before it ships.

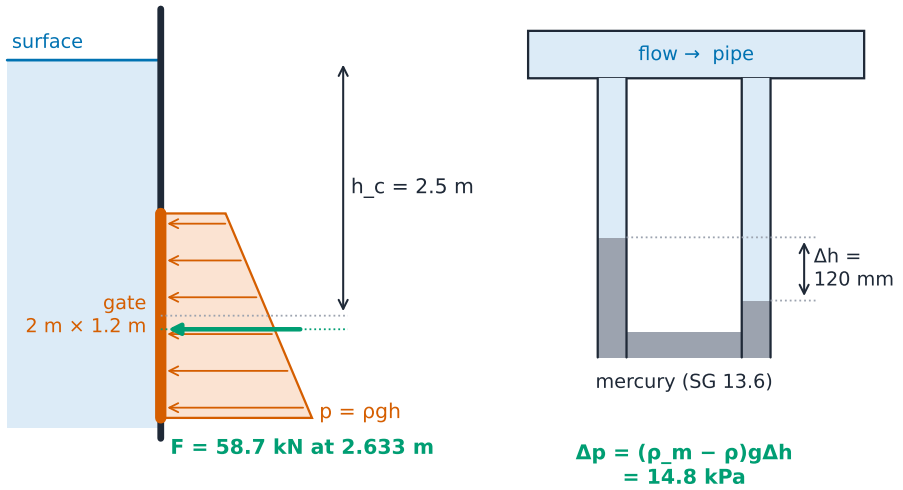
- Unit 7.1 holds the fluid still: pressure fields and the forces they put on walls.
- Units 7.2–7.3 let it move: control volumes, then Bernoulli and flow meters.
- Unit 7.4 shrinks the problem with dimensionless groups.
- Units 7.5–7.6 pay for friction: inside pipes, then around bodies.

The habit: **state the assumptions before the equation** – steady or not, friction or not, along a streamline or across one. Bernoulli is the most misused equation in engineering because people skip this step.

Fluid properties and statics

Why it matters. The loop's header tank, its access gate and every pressure gauge on the pipe are read with hydrostatics – the one part of fluid mechanics that is exact.

The idea, with a picture. A fluid at rest carries no shear, so pressure acts equally in all directions and grows with depth: $dp/dz = -\rho g$, so $p = p_0 + \rho g h$. On a plane wall the pressure forms a **pressure prism**; its resultant is $F = \rho g h_c A$ (the pressure at the centroid depth h_c times the area), and it acts at the **centre of pressure**, which is always below the centroid.



Pressure grows with depth, so the gate force acts below its centroid; a manometer turns a height into a pressure difference

Viscosity, the property statics ignores. Once the fluid moves, the **viscosity** μ sets the shear stress: $\tau = \mu\, du/dy$ (Newton's law of viscosity), with units Pa s = N s/m^2. A plate sliding at 2 m/s on a 0.5 mm oil film with $\mu = 0.29$ Pa s feels $\tau = 0.29 \times 2/0.0005 = \mathbf{1160}$ Pa. The **kinematic viscosity** $\nu = \mu/\rho$ (1.004×10^{-6} m^2/s for water) is the one that appears in Reynolds numbers.

The derivation (why the centre of pressure is lower). The moment of the pressure about the surface line is $\int \rho g y \cdot y\, dA = \rho g I_O$; dividing by $F = \rho g y_c A$ and using the parallel-axis theorem $I_O = I_c + y_c^2 A$ gives

$$y_{cp} = y_c + \frac{I_c}{y_c A}$$

The extra term shrinks as the gate goes deeper: deep gates are loaded almost uniformly.

> **Worked example 1 – the tank gate.** A vertical rectangular gate, 1.2 m wide and 2 m high, has its top edge 1.5 m below the surface. Centroid depth $h_c = 2.5$ m, $A = 2.4$ m^2: $F = 998 \times 9.81 \times 2.5 \times 2.4 = \mathbf{58.7}$ kN. With $I_c = bh^3/12 = 0.8$ m^4: $y_{cp} = 2.5 + 0.8/(2.5 \times 2.4) = \mathbf{2.633}$ m – 133 mm below the centroid, which is where the hinge or latch must react it.

> **Worked example 2 – the differential manometer.** A mercury (SG 13.6) U-tube across a pipe fitting reads $\Delta h = 120$ mm. The water above the mercury on both sides cancels except over Δh, so $\Delta p = (\rho_m - \rho) g \Delta h = (13.6 - 1) \times 998 \times 9.81 \times 0.12 = \mathbf{14.8}$ kPa. Using ρ_m alone (forgetting the water column) would overstate it by 8%.

Common mistakes. Putting the force at the centroid instead of the centre of pressure. Mixing gauge and absolute pressure in one equation. Forgetting the fluid above the manometer liquid.

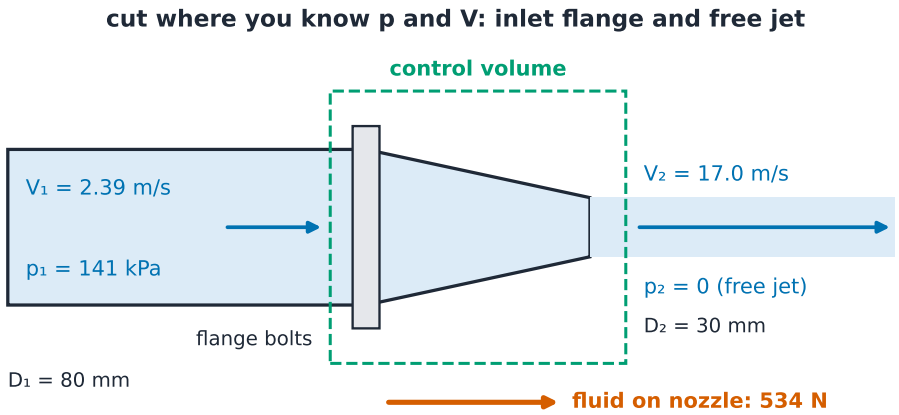
Summary. $p = p_0 + \rho g h$; $F = \rho g h_c A$ at $y_{cp} = y_c + I_c/(y_c A)$; $\Delta p = (\rho_m - \rho) g \Delta h$; $\tau = \mu\, du/dy$.

Conservation and control volumes

Why it matters. The hose nozzle on the loop's wash-down line and every pipe bend must be anchored. The anchor force comes from momentum, not from pressure alone – and the clean way to find it is a **control volume** .

The idea, with a picture. Draw a boundary, account for what crosses it. For steady flow with one inlet and one outlet:

- mass (**continuity**): $\rho V_1 A_1 = \rho V_2 A_2$, so $Q = V_1 A_1 = V_2 A_2$ for a liquid;
- momentum: $\sum \mathbf{F} = \dot{m}(\mathbf{V}_2 - \mathbf{V}_1)$, where $\sum \mathbf{F}$ includes the pressure forces on the cut faces and the force from the solid walls.



Draw the control volume where p and V are known: the inlet flange and the free jet

The derivation (where the momentum equation comes from). Newton's second law for the fluid inside the boundary, written with the Reynolds transport theorem: the rate of change of momentum inside (zero when steady) plus the net outflow of momentum $\dot{m}V_2 - \dot{m}V_1$ equals the sum of external forces. The choice of boundary is yours – cut where pressure and velocity are known, and through the bolts whose force you want.

> **Worked example 1 – the nozzle bolts.** $Q = 12$ L/s enters an 80 mm pipe and leaves a 30 mm nozzle as a free jet. $A_1 = 5.027 \times 10^{-3}$ m^2, so $V_1 = \mathbf{2.39}$ m/s; $A_2 = 7.069 \times 10^{-4}$ m^2, so $V_2 = \mathbf{17.0}$ m/s. Bernoulli from 1 to the jet ($p_2 = 0$ gauge) gives $p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) = \mathbf{141}$ kPa. The x-momentum balance on the fluid: $p_1 A_1 + F_{\text{nozzle on fluid}} = \rho Q(V_2 - V_1)$, so $F = 174.7 - 708.6 = -533.9$ N. The fluid therefore pushes the nozzle **downstream with 534 N**, and the flange bolts carry it in tension – the pressure term (709 N) wins over the jet's momentum gain (175 N).

> **Worked example 2 – a 90° bend.** The same 12 L/s turns through 90° in the 80 mm pipe at 150 kPa gauge (losses neglected, so p and V are equal at both ends). Each direction carries $pA + \rho QV = 754.0 + 28.6 = \mathbf{782.6}$ N, and the anchor must hold the resultant $\sqrt{2} \times 782.6 = \mathbf{1107}$ N at 45°. Pressure dominates at this speed – which is why bends in high-pressure, low-velocity lines still need thrust blocks.

Common mistakes. Leaving the pressure forces on the cut faces out of $\sum F$. Using gauge pressure at one face and absolute at the other. Reporting the force on the fluid when the question asks for the force on the pipe (equal and opposite).

Summary. $Q = VA$ is conserved; $\sum F = \dot{m}(V_2 - V_1)$ component by component; cut where p and V are known; the pipe feels the reverse of the force on the fluid.

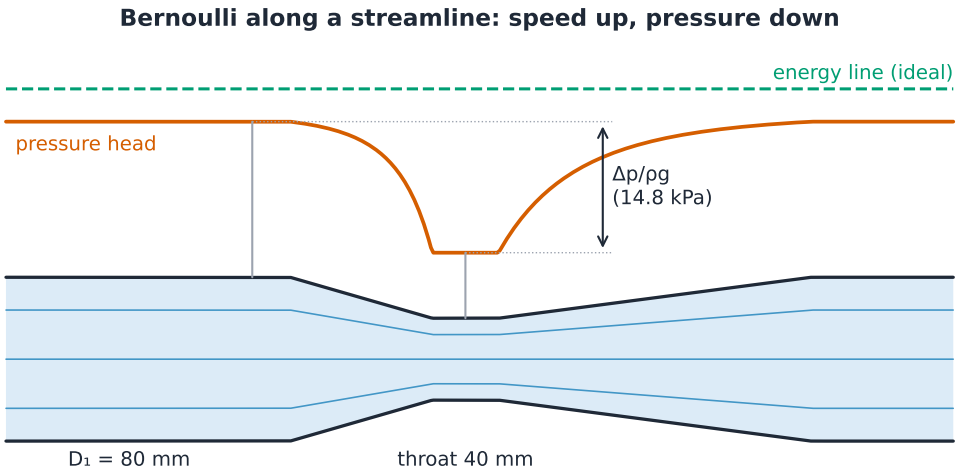
Bernoulli and flow measurement

Why it matters. The loop's flow is measured by a venturi and an air duct by a pitot tube – both are Bernoulli with a correction factor, and the factor is where the engineering is.

The idea, with a picture. Along a streamline , for steady, incompressible, frictionless flow with no pump or turbine:

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

Speed up and the pressure falls. Four assumptions – and each is a question to ask before using it.



In the throat the speed rises and the pressure head falls; the tapped difference measures the flow

The derivation (venturi equation). Continuity gives $V_1 = V_2\beta^2$ with $\beta = D_2/D_1$. Put it into Bernoulli between the taps: $\Delta p = \frac{1}{2}\rho V_2^2(1-\beta^4)$, so

$$Q = C_d A_2 \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}}$$

The **discharge coefficient** C_d (about 0.98 for a venturi, about 0.61 for a sharp orifice plate) absorbs friction and the vena contracta – the parts Bernoulli leaves out.

> **Worked example 1 – the venturi.** An 80 mm pipe narrows to a 40 mm throat ($\beta = 0.5$, $1-\beta^4 = 0.9375$); the manometer of unit 7.1 reads $\Delta p = 14.8$ kPa. Ideal: $Q_i = 1.257\times10^{-3}\sqrt{2\times14\,803/(998\times0.9375)} = \mathbf{7.07}$ L/s; measured with $C_d = 0.98$: $Q = \mathbf{6.93}$ L/s. An orifice plate with the same reading would give much less flow – the coefficient, not the equation, carries the device.

> **Worked example 2 – the pitot tube.** In an air duct the pitot-static tube reads 450 Pa between stagnation and static ports. Stagnation means $V = 0$ at the tip, so $\Delta p = \frac{1}{2}\rho V^2$ and $V = \sqrt{2\times450/1.204} = \mathbf{27.3}$ m/s. Valid while the air is effectively incompressible (Mach below about 0.3, here 0.08).

Common mistakes. Applying Bernoulli across a pump or through a region with large losses. Forgetting $1-\beta^4$ (a 3% error at $\beta = 0.5$, larger as β grows). Using $C_d = 1$ for an orifice.

Summary. $p/\rho g + V^2/2g + z$ constant along a streamline under four assumptions; meters measure Δp and convert with continuity and C_d; pitot $V = \sqrt{2\Delta p/\rho}$.

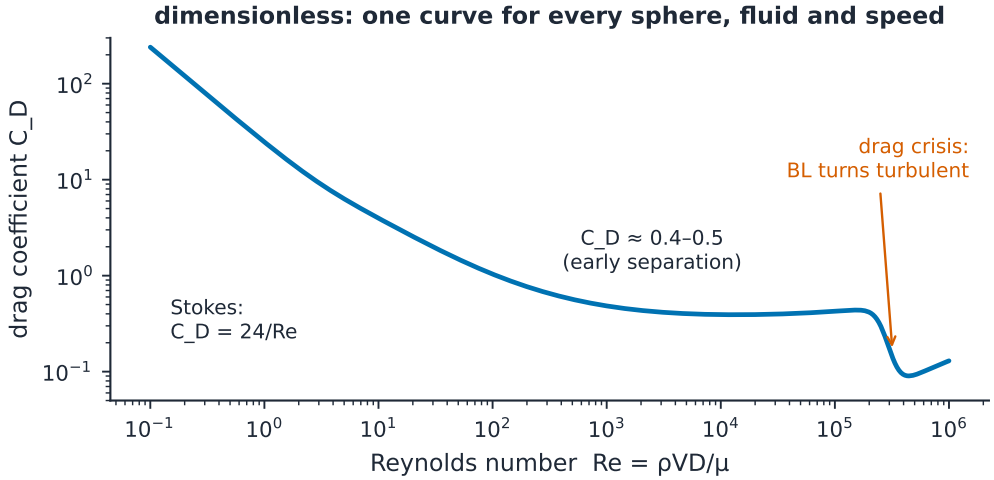
Dimensional analysis

Why it matters. Drag on a new sensor pod cannot be computed from first principles, and a full-size test is expensive. **Dimensional analysis** tells us which few groups matter, so a small, cheap test predicts the large one.

The idea, with a picture. The **Buckingham Pi theorem** π : n variables built from k base dimensions combine into $n-k$ dimensionless groups. Drag on a sphere depends on F, ρ, V, D, μ (5 variables, 3 dimensions M, L, T), so 2 groups suffice:

$$C_D = \frac{F}{\frac{1}{2}\rho V^2 A} = \phi(Re), \qquad Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

The **Reynolds number** compares inertia with viscous forces – and one measured curve then serves every sphere, fluid and speed.



Sphere drag as one dimensionless curve: C_D against Re, with the drag crisis near $Re = 3\times10^5$

The derivation (repeating variables). Choose ρ, V, D as repeating variables (they contain M, L and T between them). Make $F\rho^a V^b D^c$ dimensionless: $a = -1$, $b = -2$, $c = -2$, giving $F/(\rho V^2 D^2)$; the same for μ gives $\mu/(\rho VD) = 1/Re$. Any correct physical law must be a relation between these groups.

> **Worked example 1 – the model test.** A sensor pod, 0.5 m long, flies at 20 m/s in air ($\nu = 1.516\times10^{-5}$ m^2/s): $Re = 6.60\times10^5$. A half-scale model in a water tunnel ($\nu = 1.004\times10^{-6}$) needs the same Re (**dynamic similarity**): $V_m = 20\times(0.5/0.25)\times(1.004\times10^{-6}/1.516\times10^{-5}) = \mathbf{2.65}$ m/s – water's low ν lets the model run slowly.

Worked example 2 – scaling the force back. The model measures $F_m = 30$ N. Equal C_D means $F_p = F_m(\rho_p/\rho_m)(V_p/V_m)^2(L_p/L_m)^2 = 30 \times (1.204/998) \times (20/2.65)^2 \times 4 =$ **8.25** N. Also: the loop's pipe flow has $Re = VD/\nu = 2.39 \times 0.08/1.004 \times 10^{-6} =$ **1.90 × 10⁵** – far above 2300, so it is turbulent, which unit 7.5 needs.

Common mistakes. Matching geometry but not Re (geometric similarity alone is not enough). Matching Re and Froude number at once with the same fluid – usually impossible; choose the group that governs. Using μ where ν is meant.

Summary. $n - k$ groups; $C_D = \phi(Re)$; match Re for dynamic similarity; laminar pipe flow below about $Re = 2300$, turbulent above about 4000.

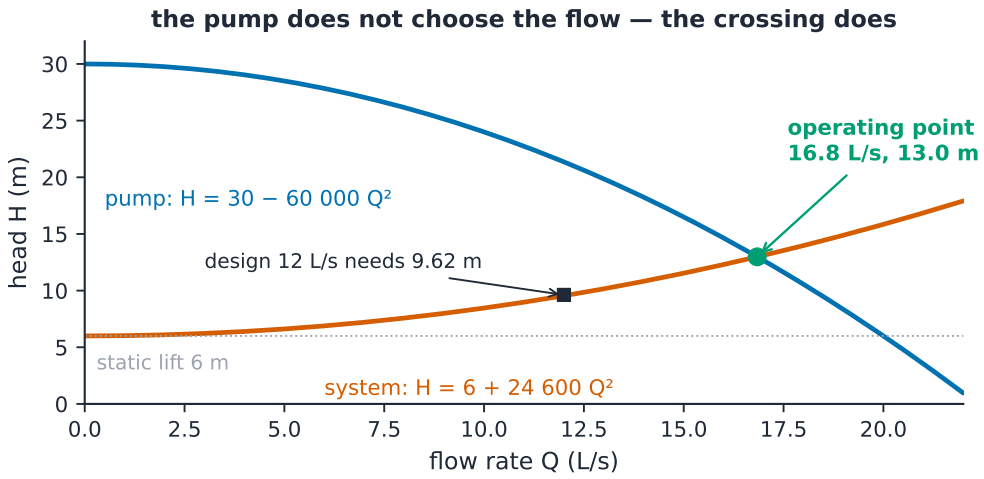
Pipe flow and losses

Why it matters. The pump in the loop must be chosen against the pipe's friction and fittings. Undersize the estimate and the loop runs short of flow; oversize it and the pump wastes power for its whole life.

The idea, with a picture. The energy equation between two points includes a **head loss** h_L:

- **major loss** (Darcy–Weisbach): $h_f = f\,(L/D)\,V^2/2g$;
- **minor loss** (fittings): $h_m = \sum K\,V^2/2g$.

The **friction factor** f is $64/Re$ for laminar flow; for turbulent flow it depends on Re and the relative roughness ε/D (Colebrook equation, or the Moody chart).



The pump curve and the system curve cross at the operating point

The derivation (why laminar $f = 64/Re$). For laminar flow the velocity profile is a parabola, and integrating it gives the Hagen–Poiseuille result $\Delta p = 32\mu LV/D^2$. Writing this as $h_f = f(L/D)V^2/2g$ forces $f = 64\mu/(\rho VD) = 64/Re$. In turbulent flow there is no such derivation – f comes from experiments, fitted by Colebrook: $1/\sqrt{f} = -2\log_{10}\big(\varepsilon/3.7D + 2.51/(Re\sqrt{f})\big)$.

Worked example 1 – laminar oil. Oil ($\nu = 1 \times 10^{-4}$ m²/s, $\rho = 870$ kg/m³) at 0.5 m/s in a 20 mm line: $Re =$ **100**, laminar, $f =$ **0.64**. Over 10 m: $h_f = 0.64 \times 500 \times 0.01274 =$ **4.08** m of oil, i.e. $\Delta p =$ **34.8** kPa.

Worked example 2 – the loop and its pump. The loop is 40 m of 80 mm commercial steel ($\varepsilon = 0.045$ mm, $\varepsilon/D = 5.63 \times 10^{-4}$), with four elbows ($K = 0.3$ each), a gate valve (0.15), entrance (0.5) and exit (1.0): $\sum K = 2.85$, and 6 m of static lift. At the design 12 L/s, $Re = 1.90 \times 10^5$ and Colebrook gives $f =$ **0.0192**; $V^2/2g = 0.290$ m, so $h_f = 0.0192 \times 500 \times 0.290 =$ **2.79** m, $h_m = 2.85 \times 0.290 =$ **0.83** m, and the system needs $H = 6 + 2.79 + 0.83 =$ **9.62** m. The pump curve is $H = 30 - 60\,000\,Q^2$. The system curve is $H = 6 + kQ^2$ with $k \approx 24\,600$ (recomputed with f at the new flow). They cross at $Q =$ **16.8** L/s and $H =$ **13.0** m – the loop runs 40% above design unless it is throttled. Water power $\rho gQH =$ **2.14** kW; at 70% pump efficiency the shaft needs **3.06** kW, inside the 5.65 kW the Course 6 shaft transmits.

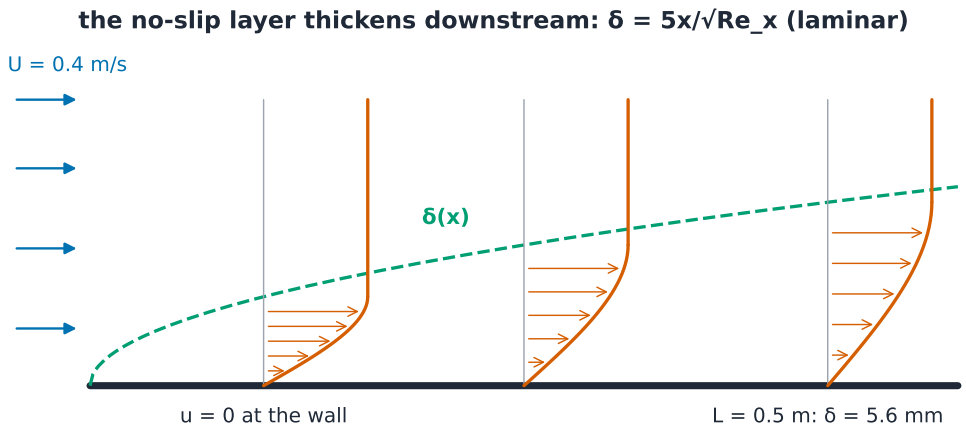
Common mistakes. Treating f as fixed when the flow changes. Reading the Moody chart with Re based on the wrong diameter. Picking a pump at the design flow without drawing the system curve (the pump runs where the curves cross, not where you wanted).

Summary. $h_f = f(L/D)V^2/2g$, $h_m = \sum KV^2/2g$; $f = 64/Re$ laminar, Colebrook turbulent; operating point = pump curve ∩ system curve; power $= \rho gQH/\eta$.

External flow

Why it matters. Wind on the loop's exposed vent pipe and the drag on any vehicle are external flows: the body sits in a stream, and a thin **boundary layer** decides the drag.

The idea, with a picture. At the wall the fluid does not slip ($u = 0$); within the boundary layer the speed rises to the free stream U. On a flat plate the laminar layer grows as $\delta = 5x/\sqrt{Re_x}$ and turns turbulent near $Re_x \approx 5 \times 10^5$. Drag has two parts: **skin friction** (shear on the surface) and **pressure drag** from **flow separation** behind blunt bodies. Both are summed into $F_D = C_D \cdot \frac{1}{2}\rho U^2 A$; lift is $F_L = C_L \cdot \frac{1}{2}\rho U^2 A$.



A laminar boundary layer thickens with distance from the leading edge

The derivation (why $\delta \propto \sqrt{x}$). Viscous diffusion spreads momentum a distance $\sqrt{\nu t}$ in time t; the fluid reaches station x after $t = x/U$, so $\delta \sim \sqrt{\nu x/U} = x/\sqrt{Re_x}$. The exact Blasius solution sets the constant to about 5.

> **Worked example 1 – the plate in water.** A plate 0.5 m long and 0.3 m wide in water at 0.4 m/s: $Re_L = \mathbf{1.99 \times 10^5}$, laminar throughout. Edge thickness $\delta = 5 \times 0.5/\sqrt{1.99 \times 10^5} = \mathbf{5.6}$ mm. Blasius $C_f = 1.328/\sqrt{Re_L} = \mathbf{0.00298}$, so the drag on one side is $0.5 \times 998 \times 0.16 \times 0.00298 \times 0.15 = \mathbf{0.036}$ N. A fully turbulent layer ($C_f = 0.074/Re_L^{0.2} = 0.0064$) would more than double it.

> **Worked example 2 – the vent pipe in wind.** A 0.1 m diameter, 3 m tall pipe in a 15 m/s wind: $Re = 15 \times 0.1/1.516 \times 10^{-5} = \mathbf{9.89 \times 10^4}$, in the range where a cylinder's $C_D \approx 1.2$ (separation dominates). Drag $F = 1.2 \times 0.5 \times 1.204 \times 225 \times 0.3 = \mathbf{48.8}$ N, and the base bending moment is $48.8 \times 1.5 = \mathbf{73.1}$ N · m – a load for the Course 6 bending check.

Common mistakes. Using frontal area for skin-friction drag (it uses wetted area) or wetted area for blunt bodies (they use frontal). Assuming C_D is constant across the drag crisis. Reading a flow picture as proof without knowing its Re.

Summary. No slip creates the boundary layer; $\delta = 5x/\sqrt{Re_x}$ laminar; $C_f = 1.328/\sqrt{Re_L}$; blunt bodies are dominated by separation; $F = C \cdot \frac{1}{2}\rho U^2 A$.