Topic 05

University Mechanical Engineering

What this course is, and how to use it

This is the fifth course in University Mechanical Engineering: **thermodynamics** – energy, entropy and the machinery that turns one into work: how to account for it, how far it can go, and how to know a claim about a machine is impossible before building it.

Scope limit, stated once: a study course, not a degree; the teaching fluids are water, air and R-134a, every cycle is checked by both laws in the course suite, and every table value used here is recomputed there from the quoted source constants.

- Units 5.1–5.3 build the accounting: state, closed systems, open systems.
- Unit 5.4 sets the limits: the second law and entropy.
- Units 5.5–5.6 spend the limits: power cycles and refrigeration.

The thermo habit: **every cycle is checked by both laws** – the suite runs the checks; the habit is looking before believing.

Properties and state

Why it matters. Before any law can be applied, the working fluid must be pinned down: what its **state** is, and which numbers move together.

The idea, with a picture. A state is a point on a **property diagram** ; two independent intensive properties fix it. On the **saturation dome** the two-phase region sits under the dome; **quality** x = vapour mass fraction walks across it.

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Left: the p–V dome, liquid to its left and vapour to its right, with one tie-line marked in quality x; right: the T–s dome of the same substance – the same two-phase region, drawn on different axes

The derivation (why two properties fix the rest). A simple compressible substance obeys a two-variable equation of state, $p = p(v, T)$, so any other equilibrium property is a function of the same pair – pick two independent ones (inside the dome, x is the walker) and everything else is determined.

> **Worked example 1 – reading a table row.** Saturated water at $T = 100\,^{\circ}\mathrm{C}$: $p_{sat} = 101.35$ kPa, $v_f = 0.001044$ m^3/kg, $v_g = 1.6729$ m^3/kg, $h_f = 419.04$ kJ/kg, $h_{fg} = 2257.0$ kJ/kg. At $x = 0.5$: $v = v_f + x(v_g - v_f) = 0.837$ m^3/kg, $h = h_f + x\,h_{fg} = 1547.5$ kJ/kg – the linear-in-x rule holds for v, u, h, s because dome properties differ only by the latent term.

> **Worked example 2 – ideal-gas territory.** Air at 300 K, 101.325 kPa: $pv = RT$ with $R = 0.287$ kJ/(kg K) gives $v = RT/p = 0.8497$ m^3/kg. At 2× that pressure, same T: $v = 0.4248$ m^3/kg – exactly half: the isotherm is a hyperbola (suite-checked both ways).

Common mistakes. Using v averaged by volume instead of mass (quality is a mass fraction). Reading a superheated row from a saturation column. Treating $pv = RT$ inside the dome (ideal gas ends where condensation begins).

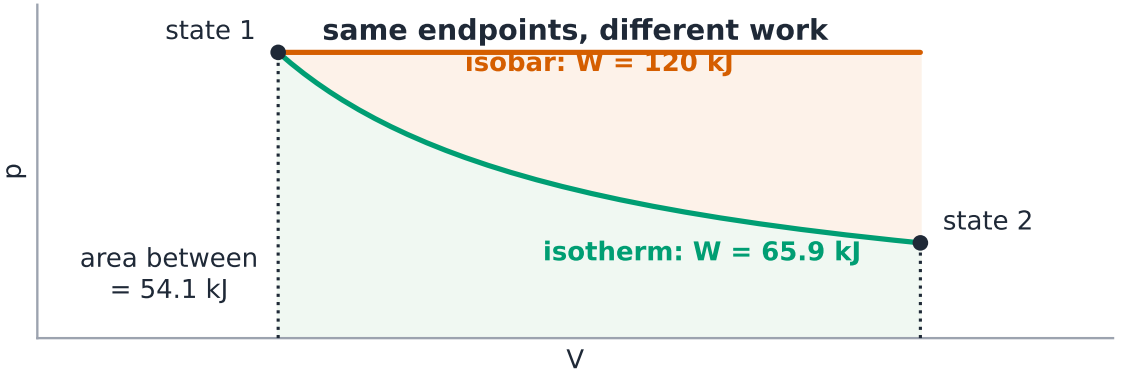
Summary. Two independent properties fix the state; the dome needs x as the walker; linear-in-x between f and g; $pv = RT$ only outside the dome.

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First law for closed systems

Why it matters. The first law is the accountant: energy changes by work and heat crossing the boundary, nothing else, and a closed system is the simplest ledger to keep.

The idea, with a picture. $\Delta E = Q - W$ (work **by** the system positive). On a **p–V diagram** the **boundary work** $W = \int p\,dV$ is the area under the path – so the work depends on the path, not just the endpoints.



Two paths between the same states: the area under each is that path's work – path dependence made visible

The derivation (why work is path-dependent). $W = \int_1^2 p\,dV$ integrates the path $p(V)$ itself, and different connective paths trace different curves between the same endpoints – only U (and H, S) are state functions, the whole reason a cycle's net $\oint \delta W = \oint \delta Q$ closes.

> **Worked example 1 – three paths, one pair of states.** Air from $(200\ \text{kPa}, 0.3\ \text{m}^3)$ to $(200\ \text{kPa}, 0.9\ \text{m}^3)$ versus via an isotherm through the same ends. At constant p: $W = p\Delta V = 200 \times 0.6 = 120$ kJ. Isothermal from $v_1 = v$ to $3v$: $W = mRT\ln(v_2/v_1) = p_1V_1 \ln 3 = 200{\times}0.3{\times}1.0986 = 65.9$ kJ – same endpoints, 54 kJ apart, the area between the curves is exactly the difference (suite-asserted).

> **Worked example 2 – the adiabatic check.** Same start, compressed adiabatically ($Q = 0$) to 0.15 m^3: $pV^\gamma = \text{const}$ with $\gamma = 1.4$ gives $p_2 = 200 \times (0.3/0.15)^{1.4} = 527.8$ kPa; $W = (p_2V_2 - p_1V_1)/(1-\gamma) = (79.2 - 60)/(1 - 1.4) = 48$ kJ of work **on** the gas; $\Delta U = -W = +48$ kJ – the ledger closes.

Common mistakes. Sign errors on W (by vs on). Using $p_1V_1 \approx mRT$ with V in litres and R in kJ. Forgetting that $\Delta U = mc_v\Delta T$ holds for ideal gas even when the process is not constant-volume – it is a property relation.

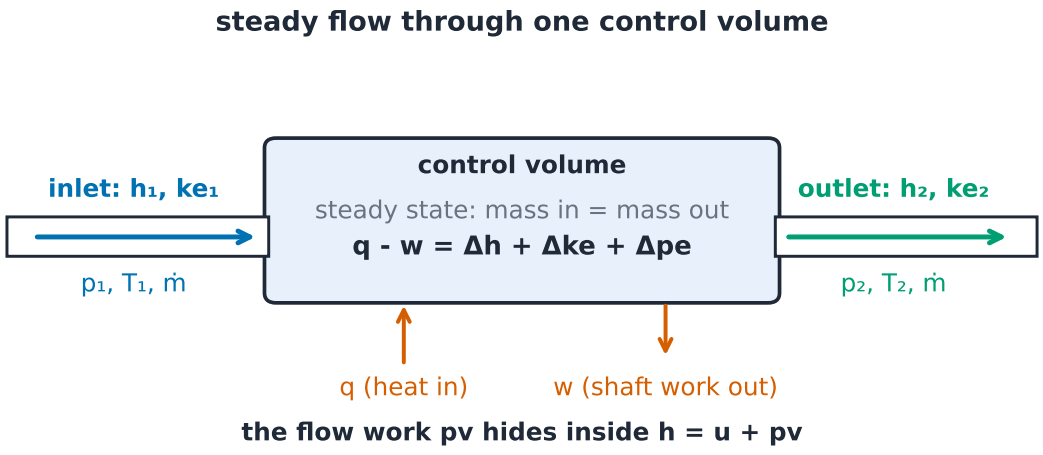
Summary. $\Delta U = Q - W$; boundary work is area under the path; state functions versus path functions; the ideal-gas toolkit ($pv = RT$, c_v, c_p, pV^γ for reversible adiabatic).

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First law for open systems

Why it matters. Real machines are pipes, not pistons: mass flows through a **control volume** and carries energy with it, and steady-state analysis is where turbines, pumps and nozzles live.

The idea, with a picture. The steady-flow energy equation per inlet/outlet port: $q - w = \Delta h + \Delta ke + \Delta pe$. The flow work pv hides inside $h = u + pv$.



One inlet, one outlet, steady state: enthalpy in, enthalpy out, shaft work and heat at the ports

The derivation (where pv goes). Pushing 1 kg of fluid into the control volume costs pv (pressure times volume) of flow work; the same term reappears pushing it out. Combining that transport term with internal energy gives enthalpy – which is why the open-system ledger runs on h, not u: the bookkeeping absorbed the flow work.

> **Worked example 1 – a turbine stage.** Steam enters at 3 MPa, 400 °C ($h_1 = 3230.9$ kJ/kg, $s_1 = 6.9212$ kJ/(kg K)) and expands adiabatically to 50 kPa. Ideal (**isentropic**): $s_2 = s_1$ gives $x_2 = (6.9212 - 1.0910)/6.5029 = 0.89655$, so $h_{2s} = h_f + x\,h_{fg} = 340.49 + 0.89655 \times 2305.4 = 2407.4$ kJ/kg; $w_s = h_1 - h_{2s} = 823.5$ kJ/kg. With $\eta_t = 0.85$: $w = 0.85 \times 823.5 = 700.0$ kJ/kg, $h_2 = 3230.9 - 700.0 = 2530.9$ – the suite recomputes both legs.

> **Worked example 2 – a pump and a nozzle.** Pump lifts liquid water 0.5 MPa → 3 MPa: $w_p \approx v\,\Delta p = 0.001005 \times 2500 = 2.5$ kJ/kg. Nozzle accelerates air 10 → 300 m/s adiabatically, no work: $\Delta h = -\Delta ke = -(300^2 - 10^2)/2 = -44.95$ kJ/kg, $h_2 = h_1 - 44.95$. Three devices, one equation, the ports carry the ledger.

Common mistakes. Dropping ke for a nozzle (it is the point of the device) or keeping it for a pump (negligible there). Using $c_p \Delta T$ for the exit of a two-phase expansion (inside the dome h is not $c_p \Delta T$ from a superheated start). Forgetting $\dot{W} = \dot{m}\,w$ for the power answer.

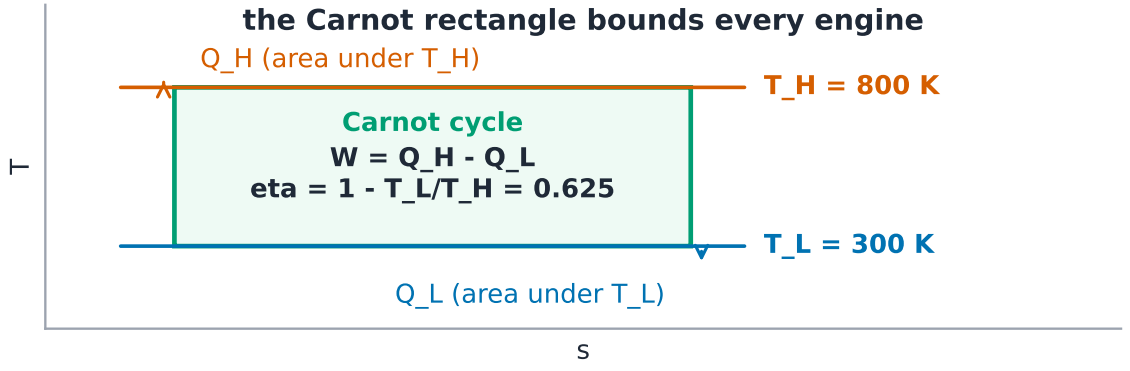
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Summary. $q - w = \Delta h + \Delta ke + \Delta pe$; enthalpy absorbs the flow work; ke matters where velocity is the product; isentropic then efficiency is the two-step turbine method.

Second law and entropy

Why it matters. The first law only balances; the second law forbids. Entropy is the accounting quantity of the forbidden, and the Carnot bound stops bad machine claims on paper.

The idea, with a picture. $ds = \delta q_{rev}/T$ – **entropy** ; $\sigma \geq 0$ for every process, $= 0$ only for reversible ones. On a **T–s diagram** T–s the reversible heat is area under the curve, and the $\eta_C = 1 - T_L/T_H$ appears as the rectangle ratio.



Heat as area on the T–s plane; the Carnot rectangle between T_H and T_L bounds every engine

The derivation (Kelvin–Planck to Carnot). No cycle converts heat from one reservoir fully into work; so for a reservoir pair the best cycle must reject the least, and a reversible one must be the best (otherwise running it backwards would break the law). The reversible pair of isotherms and adiabats – Carnot – then has $Q_H/T_H = Q_L/T_L$ exactly, giving $\eta_C = 1 - T_L/T_H$. Between 800 K and 300 K: $\eta_C = 0.625$; a claimed 0.70 engine between them is impossible, no prototype needed.

> **Worked example 1 – entropy balance on an irreversible adiabatic valve.** Steam at 1000 kPa, $h = 3059$ kJ/kg, $s = 7.155$ kJ/(kg K) throttled to 200 kPa ($h_2 = h_1$): at 200 kPa, $h = 3059$ is superheated, $s_2 = 7.289$ kJ/(kg K); $\sigma = s_2 - s_1 = 0.134$ kJ/(kg K) > 0 – irreversible with no heat transfer at all.

> **Worked example 2 – the Carnot ledger.** A Carnot engine takes 1000 kJ from 800 K, rejects to 300 K: $W = 625$ kJ, $Q_L = 375$ kJ, $\Delta S_{hot} = -1.250$, $\Delta S_{cold} = +1.250$: universe unchanged – reversible. Burn the same fuel irreversibly at 2000 K before transferring: $\sigma = Q(1/800 - 1/2000) = 0.75$ kJ/K – the lost work $T_0\sigma = 300 \times 0.75 = 225$ kJ, exactly the exergy destroyed.

Common mistakes. $ds = \delta q/T$ applied to an irreversible path (it needs q_{rev}, or

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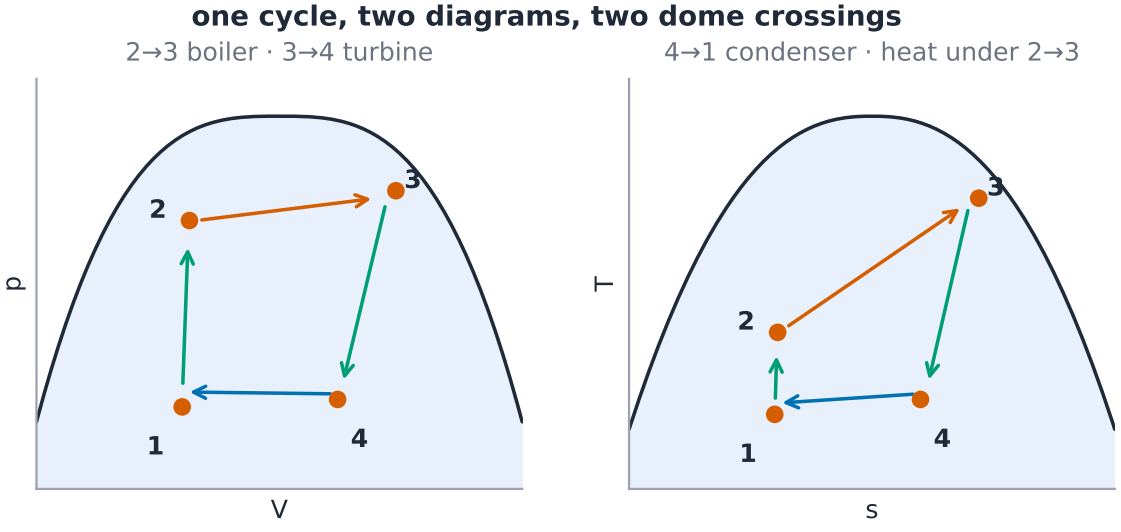
use the entropy balance with σ). Reading efficiency claims without the reservoir temperatures. Taking s constant across a throttling (throttle keeps h, not s).

Summary. $ds = \delta q_{rev}/T$; $\sigma \geq 0$; Carnot bounds by reservoir ratio; lost work $= T_0\sigma$; T–s area = reversible heat.

Power cycles

Why it matters. The power plant is where the two laws pay rent: the Rankine cycle converts fuel to shaft work through phase change, and its efficiency is a battlefield of boiling pressure, superheat and reheat.

The idea, with a picture. The basic **Rankine cycle** : pump → boiler → turbine → condenser, drawn on both p–V and T–s. Heat in at the high-pressure isobar, work out at the isentropic expansion, heat out at the condenser, work in at the pump.



The same Rankine cycle on both diagrams: the dome crossings at the turbine exhaust and the pump inlet are what make it a two-phase machine

The derivation (why the boiler pressure raises efficiency). The cycle heat-in average temperature rises with boiler pressure, and the second-law ceiling $\eta \leq 1 - T_{out}/T_{in,avg}$ rises with it. Meanwhile the pump work grows only as $v\Delta p$ (liquid v is tiny), so the gain outruns the cost until metallurgy stops it. The suite demonstrates: the same cycle at 1 MPa versus 4 MPa moves η from 25.3% to 32.3% with the same T_{max}.

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Worked example 1 – the ideal Rankine baseline. Boiler at 4 MPa, saturated vapour ($h_3 = 2801.4$ kJ/kg, $s_3 = 6.0700$); condenser at 10 kPa. Turbine: $s_4 = s_3$ gives $x_4 = (6.0700 - 0.6492)/7.5009 = 0.7227$, $h_4 = 191.81 + 0.7227 \times 2392.8 = 1921.1$. Pump: $h_2 = h_1 + v_f(p_2 - p_1) = 191.81 + 0.001010 \times 3990 = 195.8$. Work out $2801.4 - 1921.1 = 880.3$, work in 4.0; heat in $2801.4 - 195.8 = 2605.6$; $\eta = 876.3/2605.6 = 33.6\%$ – every step suite-verified.

Worked example 2 – two upgrades, priced. Superheat to 4 MPa, 480 °C ($h_3 = 3398.3$, $s_3 = 6.7677$): $x_4 = 0.8157$, $h_4 = 2143.6$; η rises to 39.1%, and the exhaust quality climbs from 0.72 toward 0.82 – better, though still wet: real plant runs superheat **and** reheat to stay dry. Reheat from an intermediate 800 kPa back to 480 °C pushes the exhaust above 0.9, clearing the 0.88 blade-erosion line.

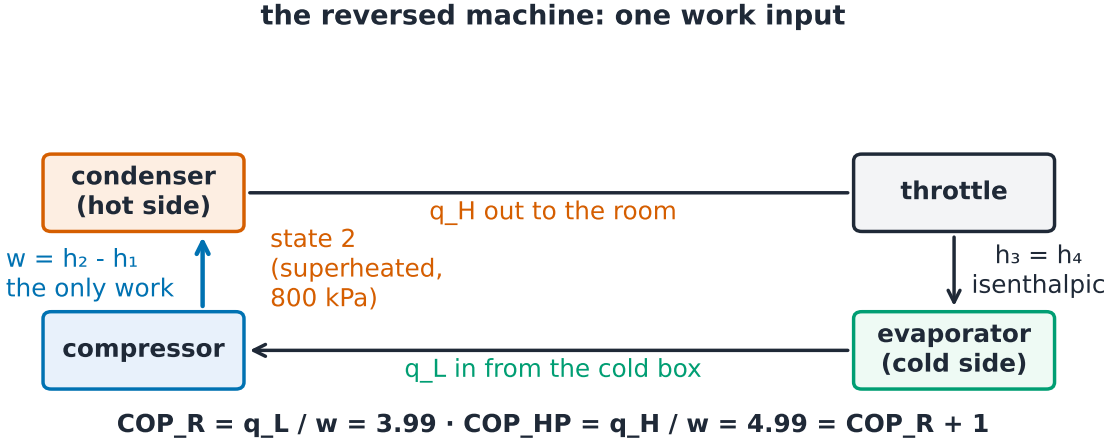
Common mistakes. Subtracting h_1 instead of h_2 in the heat-in term (the pump outlet is where heat starts). Setting $s_2 = s_1$ across the pump (it nearly is, but use $h_2 = h_1 + v\Delta p$). Ignoring exhaust quality – a fine cycle can still ruin blades.

Summary. Four components, two dome crossings; isentropic turbine, liquid pump; superheat and reheat buy efficiency and dry exhaust; boiler pressure raises T_{avg} in.

Refrigeration and heat pumps

Why it matters. Running the cycle backwards moves heat uphill – one skeleton serves the refrigerator and the heat pump alike.

The idea, with a picture. The **vapor-compression cycle** : compressor → condenser → throttle → evaporator. Performance is $\mathrm{COP}_R = q_L/w_{in}$ or $\mathrm{COP}_{HP} = q_H/w_{in}$. The reversed-**Carnot** bounds are $\mathrm{COP}_R \le T_{cold}/(T_{hot} - T_{cold})$ for cooling and $\mathrm{COP}_{HP} \le T_{hot}/(T_{hot} - T_{cold})$ for heating. Use absolute temperatures in kelvin.



The reversed Rankine: condenser above the dome, throttle replacing the pump, evaporator below – the compressor is the only work input

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The derivation (why the throttle, not a turbine). The expansion could recover work – but the flow is wet and two-phase turbines are not worth it; a throttle is cheap, isenthalpic, destroys only a few kJ/kg, and the compressor simply re-pays them.

> **Worked example 1 – an R-134a refrigerator.** Evaporator $-10\,^{\circ}\mathrm{C}$ ($h_1 = 394.6$ kJ/kg, $s_1 = 1.729$ kJ/(kg K)); compressor to 800 kPa isentropically: $h_2 = 432.9$ (superheated). Condenser exit saturated liquid $h_3 = 241.8$; throttle keeps $h_4 = 241.8$. Then $q_L = h_1 - h_4 = 152.8$, $w_{in} = 38.3$, $\mathrm{COP}_R = 152.8/38.3 = 3.99$. Reversed-Carnot between -10 and $31.3\,^{\circ}\mathrm{C}$: bound $= 263.2/(304.5 - 263.2) = 6.37$ – real COP sits below, as it must.

> **Worked example 2 – the same machine heating.** As a heat pump, $q_H = h_2 - h_3 = 432.9 - 241.8 = 191.1$ and $\mathrm{COP}_{HP} = 191.1/38.3 = 4.99 = \mathrm{COP}_R + 1$ ✓ – the same numbers read at different ports (suite-asserted).

Common mistakes. $\mathrm{COP} = w/q$ (inverted – COP is benefit over cost). Applying the reversed-Carnot bound with temperature differences instead of absolute temperatures. Reading h_2 from the saturated-vapour row (the exit is superheated).

Summary. Reverse the skeleton, swap turbine for throttle; the compressor is the only work; COP is benefit over cost under the reversed-Carnot ceiling, with $\mathrm{COP}_{HP} = \mathrm{COP}_R + 1$.

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