

Topic 04

University Mechanical Engineering

What this course is, and how to use it

This is the fourth course in University Mechanical Engineering: **engineering materials** – why materials behave as they do, how that behaviour is measured, and how a material is chosen, processed and certified for a part.

The running case closes a loop opened in course 3: the till mount was a spring of $k = 4900 \text{ N/m}$ – this course asks what it is *made of*. Four candidates are followed throughout: **low-carbon steel** low-carbon steel, **aluminium alloy 6061**, **cast nylon 6** and a **carbon-fibre/epoxy laminate**. Every number in this handout was computed for this course – the elasticity chains recompute, the phase fractions sum to one, the probability curves integrate, and the selection index ranks the candidates.

- Units 4.1–4.2 are why and how much; 4.3–4.4 the families; 4.5–4.6 the making and the surviving.

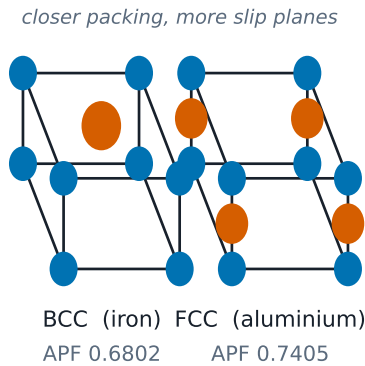
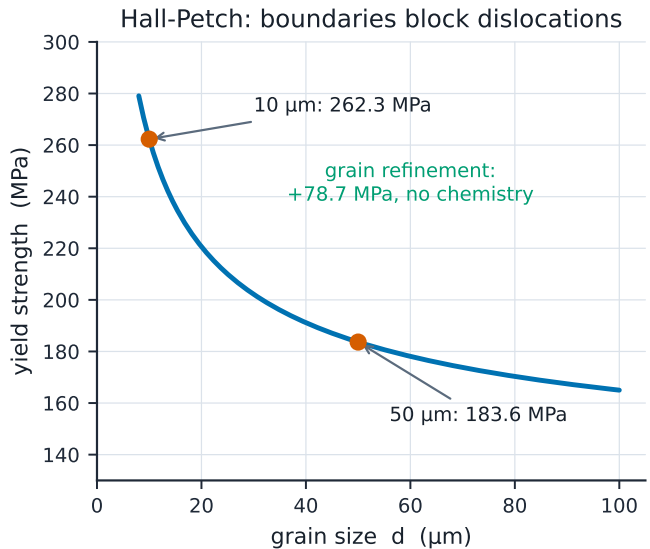
Each unit: the question, the machinery, two escalating worked examples checked a second way, the common mistakes, a summary.

Structure and bonding

Why it matters. The mount's stiffness, yield and failure all trace to atoms and microstructure. Choosing blind is choosing by rumour.

The idea, with a picture. Bonding type sets the baseline: metallic bonds give ductile conductors (steel, aluminium), covalent networks give stiff insulators (ceramics), polymer chains give light viscoelastic solids (nylon). Above the bond, the **crystal structure** packs atoms: body-centred cubic and face-centred cubic differ in density of packing – and packing plus defects sets how a metal yields.

Two packings of iron and aluminium



*Denser packing, finer grains: structure writes the property

The definitions. The **atomic packing factor** is the fraction of a unit cell filled by atoms treated as hard spheres: $\pi\sqrt{3}/8 = \mathbf{0.6802}$ for BCC (iron at room temperature) and $\pi/(3\sqrt{2}) = \mathbf{0.7405}$ for FCC (aluminium). A **dislocation** is a line defect whose motion is plastic flow; a **grain boundary** is a wall between crystals that blocks dislocations – smaller grains mean more walls. **Grain refinement** strengthens by that road: the Hall-Petch relation $\sigma_y = \sigma_0 + k/\sqrt{d}$.

The derivation. The bracket steel: $\sigma_0 = 120$ MPa and $k = 0.45$ MPa $\cdot \sqrt{\text{m}}$. At $d = 50$ μm : $\sigma_y = 120 + 0.45/\sqrt{50 \times 10^{-6}} = 120 + 63.6 = \mathbf{183.6}$ MPa ✓; refine to $d = 10$ μm : $120 + 142.3 = \mathbf{262.3}$ MPa ✓ – a gain of **78.7** MPa from processing alone, no chemistry changed ✓ (the two points are the picture's right panel).

Worked example 1 – read the packing. Aluminium is FCC, iron is BCC: the APF ratio is $0.7405/0.6802 = 1.089$ – aluminium's lattice fills 8.9% more of space with equally sized atoms, the geometric root of its lower density per atom and its many slip planes: FCC metals form and deform easily (aluminium foil), BCC iron needs warmth to match it.

Worked example 2 – escalate to the mount's bracket. The specification demands $\sigma_y \geq 250$ MPa with 20% elongation. Hall-Petch says the grain size must satisfy $120 + 0.45/\sqrt{d} \geq 250$, so $d \leq (0.45/130)^2 = \mathbf{1.20 \times 10^{-5} \text{ m} = 12.0 \mu\text{m}}$ ✓ – grain refinement alone reaches the target (10 μm gives 262.3), and the elongation is barely touched because boundaries block dislocations without embrittling the way carbon does.

Common mistakes. Treating APF as density (it is a geometric ratio; atomic mass still decides ρ). Quoting Hall-Petch outside its measured material and grain-size range (other mechanisms can dominate at nanometre grain sizes). Forgetting that disloca-

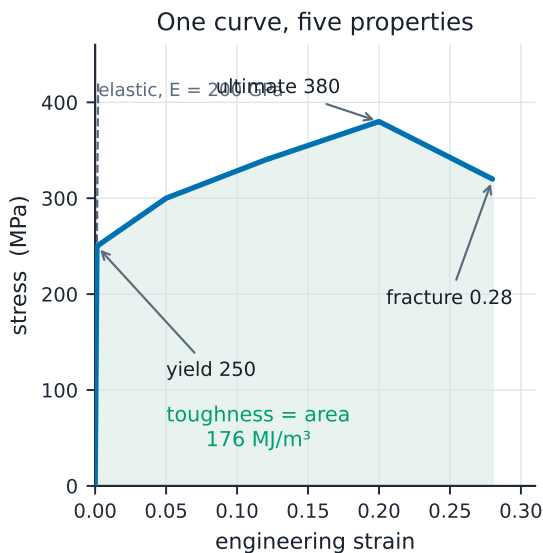
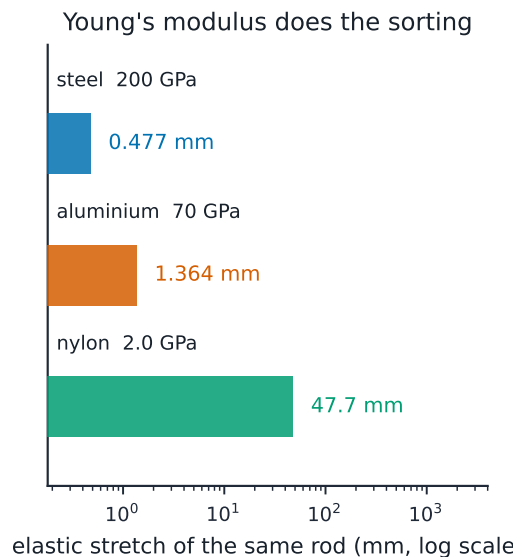
tions are the *mechanism* of ductility, not a defect to eliminate.

Summary. Bonding sets family behaviour; packing: BCC 0.6802, FCC 0.7405. Dislocations glide, boundaries block: $\sigma_y = \sigma_0 + k/\sqrt{d}$. The bracket steel: $183.6 \rightarrow 262.3$ MPa for $50 \rightarrow 10 \mu\text{m}$.

Stress-strain and elastic behaviour

Why it matters. Certification numbers – stiffness, yield, toughness – are read from one curve, and the test that produced it has conventions that make or break comparisons.

The idea, with a picture. The mount rod test: a 10.0 mm rod, 0.50 m long, pulled to 15.0 kN. Steel stretches elastically by less than half a millimetre; the same load in aluminium or nylon is a different story.



*One curve, five properties - and E sorts the elastic league

The definitions. **Engineering stress** is $\sigma = F/A_0$ on the *original* area; **true stress** uses the current area. For uniform uniaxial deformation at constant volume, before necking, $\sigma_t = \sigma(1 + \varepsilon)$. For uniform extension, true strain is $\varepsilon_t = \ln(1 + \varepsilon)$. **Young's modulus** E is the elastic slope: 200 GPa (steel), 70 GPa (aluminium), 2.0 GPa (nylon). The **yield strength** is where plastic flow begins (0.2% offset convention); **toughness** is the *area* under the curve – energy to fracture, in MJ/m³; **ductility** is strain at fracture.

The derivation. The rod: $A_0 = \pi(0.010)^2/4 = 7.854 \times 10^{-5} \text{ m}^2$, $\sigma = F/A_0 = \mathbf{191.0}$ MPa ✓ – below the steel's 250 MPa yield, so purely elastic: $\varepsilon = \sigma/E = 9.55 \times 10^{-4}$, stretch = $\varepsilon L = \mathbf{0.477}$ mm ✓. Aluminium at the same load: $191.0/70\,000 = 2.73 \times 10^{-3}$, stretch $\mathbf{1.364}$ mm – $2.86\times$ more, exactly the modulus ratio $200/70$ ✓ (the second-way check: deflections scale inversely with E). Nylon: $191.0/2000 = 9.55 \times 10^{-2} \rightarrow 47.7$

mm – off the elastic map, which is why the mount's stops exist. True vs engineering at $\varepsilon = 0.20$: $\varepsilon_t = \ln 1.20 = \mathbf{0.1823}$, $\sigma_t = 1.20\sigma$ – the two languages diverge exactly when necking starts to matter ✓. Toughness of the bracket steel (area of the curve in the picture, trapezoid over the marked points): **176 MJ/m³** ✓.

Worked example 1 – the 0.2% offset, done honestly. A supplier's plot shows no sharp yield. Draw the elastic line, shift it 0.002 in strain; where it crosses the curve is σ_y . The certificate reads 250 MPa – but for a *different* batch the same convention gave 310: the convention, not the material, changed? No – check the gauge length. Ductility percentages only compare at equal gauge length, because necking localises: the report without gauge length is a rumour.

Worked example 2 – escalate to toughness budgets. The drop test of course 3 delivers 47.56 J to the mount. A bracket volume of $47.56/176 = \mathbf{0.270}$ cm³ of this steel could absorb the whole blow plastically – the real bracket is 4.0 cm³, so the drop rides at 6.8% of its energy capacity ✓ (two courses tie: course 3's joules, this course's area-under-curve).

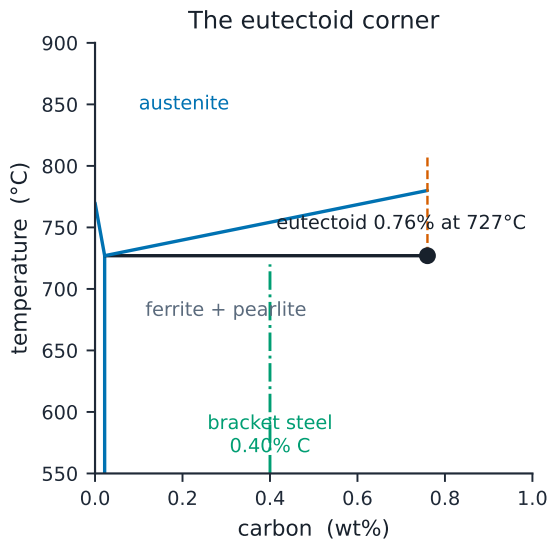
Common mistakes. Mixing engineering and true stress across a table (state the column). Reading yield off a curve with no offset convention stated. Treating toughness (area) as strength (height) – one is joules, the other pascals.

Summary. $\sigma = F/A_0$; E steel/aluminium/nylon = 200/70/2.0 GPa; the rod: 191.0 MPa, 0.477 mm vs 1.364 mm. True = $\ln(1 + e)$, 1.2σ at $e = 0.20$. Toughness 176 MJ/m³ – the joules budget.

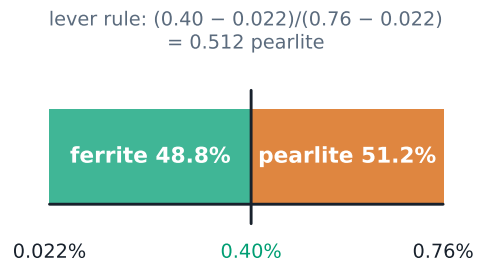
Metals and alloys

Why it matters. The steel bracket's final properties are written by chemistry and thermal history – the same steel ships at four different strength/ductility pairs.

The idea, with a picture. The iron-carbon **phase diagram** reads composition and temperature into phases; just below 727°C, the eutectoid reaction turns one austenite grain into lamellar pearlite – and how fast you cool past that line decides everything.



Just below 727°C: the tie-line weighed



*Read the phases off the diagram; weigh them with the lever rule

The definitions. The **eutectoid** point (0.76 wt% C, 727°C) is where austenite decomposes to **pearlite** (ferrite + cementite lamellae). Slow cooling gives coarse pearlite (soft), quenching bypasses pearlite to **martensite** (hard, brittle, must be tempered). The **lever rule** weighs phase fractions on a tie-line. **Strain hardening** raises strength by cold work: $\sigma = K\varepsilon^n$. Alloying adds **precipitates** that pin dislocations (the 6061 aluminium's Mg_2Si).

The derivation. The bracket steel at 0.40 wt% C, just below 727°C: pearlite fraction $= (C_0 - C_\alpha)/(C_e - C_\alpha) = (0.40 - 0.022)/(0.76 - 0.022) = \mathbf{0.512}$, ferrite **0.488** – they sum to one ✓ (the picture's right panel is exactly this tie-line). Cold work: with $K = 530$ MPa and $n = 0.20$, a 25% cold reduction carries true stress $530 \times 0.25^{0.20} = \mathbf{401.7}$ MPa – the annealed 250 MPa becomes a worked 420 MPa *engineering* yield, but elongation falls 28% → 12%.

Worked example 1 – pick the state, not just the grade. The mount bracket needs formability in production and strength in service. Buy the steel annealed (250 MPa, 28% elongation), form it, then work-harden or heat-treat to the service state – the certificate names *both* states, because the part that ships is the processed one.

Worked example 2 – escalate to the quench window. The eutectoid temperature sets the heat-treat ceiling: austenitise above 727°C (our 0.40% steel: into the full-austenite field), quench fast enough to miss the nose of the transformation curve, temper to trade hardness back for toughness. The 200°C temper ships 950 MPa at 8% elongation; the 600°C temper ships 620 MPa at 16% (the full menu is unit 4.5's picture) – the certification picks one line of that menu.

Common mistakes. Reading phases above the diagram's region (ferrite stops at 0.022% below 727°C). Adding strength mechanisms arbitrarily – they interact, so the

final certificate is measured, not summed. Confusing cementite (a phase) with pearlite (a mixture).

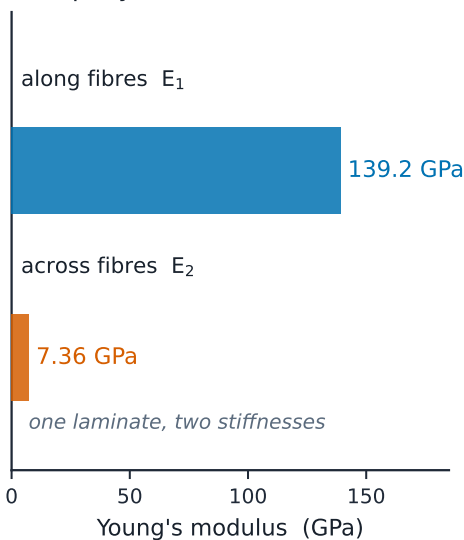
Summary. Eutectoid 0.76% C at 727°C; 0.40% steel just below: 51.2% pearlite + 48.8% ferrite. $\sigma = K\varepsilon^n$: 401.7 MPa true at 25% work. The state ships, not the grade.

Polymers, ceramics and composites

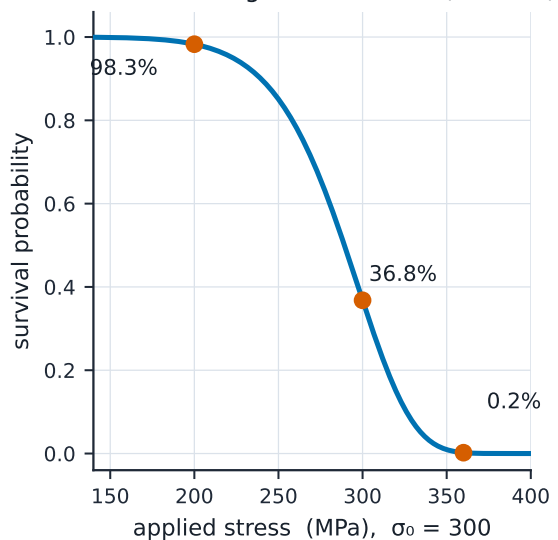
Why it matters. Half the mount's competitors are not metals: nylon promises silence and lightness, ceramic promises stiffness, and the laminate promises both – with catches that are the unit.

The idea, with a picture. The laminate's two moduli – along and across the fibres – and the ceramic's strength statistics: two pictures that carry the whole non-metal story.

Carbon-epoxy, $V_f = 0.60$: 18.9× anisotropy



Ceramic strength is statistics ($m = 10$)



*Fibres are one-way promises; ceramics are statistics

The definitions. A **composite** combines phases across a visible scale; the **rule of mixtures** averages properties by volume fraction. The equal-strain model estimates longitudinal stiffness by a weighted arithmetic mean; the equal-stress model gives a weighted harmonic mean. These idealised models provide bounds, not an exact general transverse modulus. **Viscoelasticity** is the polymer's time-dependent stiffness, governed by the **glass transition** (nylon 6: $T_g = 55^\circ\text{C}$). A **ceramic** carries few slip systems: strong in compression, brittle in tension, strength scattered – the **Weibull modulus** m prices the scatter: survival $P_s = \exp[-(\sigma/\sigma_0)^m]$.

The derivation. The laminate, $V_f = 0.60$, $E_f = 230$ GPa, $E_m = 3.0$ GPa: along the fibres $E_1 = 0.60 \times 230 + 0.40 \times 3.0 = \mathbf{139.2}$ GPa ✓; across them $E_2 =$

$E_f E_m / (V_f E_m + V_m E_f) = 690 / 93.8 = \mathbf{7.36}$ GPa ✓ – an anisotropy of **18.9**, one material with two stiffnesses. The ceramic, $m = 10$, $\sigma_0 = 300$ MPa: $P_s(200) = e^{-(2/3)^{10}} = \mathbf{0.983}$, $P_s(300) = e^{-1} = \mathbf{0.368}$, $P_s(360) = e^{-1.2^{10}} = \mathbf{0.002}$ – twenty per cent over the characteristic strength erases 98% of the survivors: brittle design is statistics.

Worked example 1 – the nylon mount in the festival sun. The booth reaches 45°C, close to nylon's $T_g = 55^\circ\text{C}$: the modulus sags viscoelastically (a factor 2–3 is typical across the transition) and creep under sustained load begins. The certification therefore thermal-cycles the nylon candidate to 55°C and re-reads the spring rate – a mount that is stiff in the morning and soft by afternoon fails the till's 2 mm clearance, whatever the room-temperature datasheet says.

Worked example 2 – escalate to the laminate bracket. Loaded along the fibres, 139.2 GPa at $\rho = 1.60$ Mg/m³ beats everything in the case; loaded across, 7.36 GPa is nylon-class. The design answer is ply orientation: a $[0/\pm 45/90]$ lay-up trades some axial stiffness for usable transverse and shear response – the certificate quotes a *laminate* modulus per direction, never a single number.

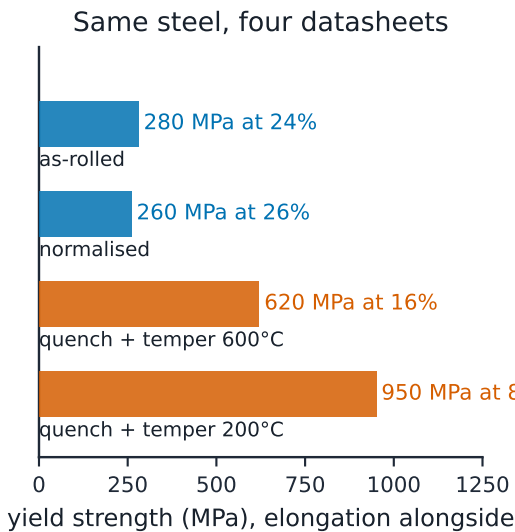
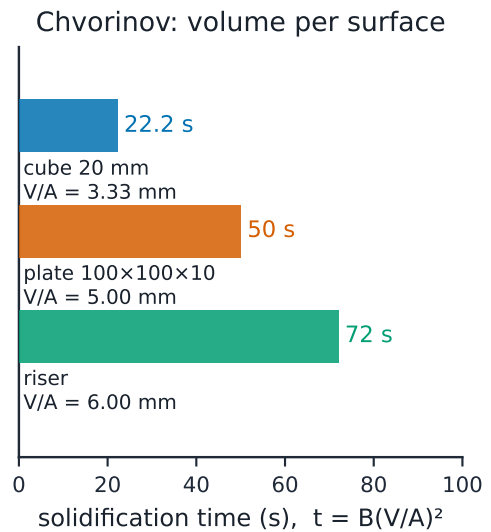
Common mistakes. Using E_1 where the load crosses the fibres (the 19-fold error). Treating the rule of mixtures as exact for strength (fibres fail statistically, not in unison). Designing ceramics to a mean strength instead of a survival probability.

Summary. $E_1 = 139.2$ GPa, $E_2 = 7.36$ GPa, anisotropy 18.9; $T_g(\text{nylon}) = 55^\circ\text{C}$ – check the sun. Ceramic survival $e^{-(\sigma/\sigma_0)^m}$: 0.983 / 0.368 / 0.002 at 200 / 300 / 360 MPa.

Processing and property links

Why it matters. The menu of bracket properties is written by thermal and mechanical processing; the buyer who ignores it buys a rumour.

The idea, with a picture. Four heat-treatment routes for the same bracket steel, and the casting rule that times solidification: the process map is the property map.



*The furnace writes the datasheet - and the riser sizes the casting

The definitions. Heat treatment – **quenching** from the austenite field to martensite, then **tempering** to give toughness back; the temper temperature dials the trade-off. **Casting** freezes liquid into shape; **solidification** time follows Chvorinov's rule $t = B(V/A)^2$ – volume over cooling surface area, all squared. Work hardening (unit 4.3) is the mechanical member of the same family.

The derivation. The four routes, same 0.40% steel: as-rolled **280** MPa at 24% elongation; normalised **260** at 26%; quenched + tempered 600°C **620** at 16%; quenched + tempered 200°C **950** at 8% ✓ (the picture's left panel – strength triples as ductility halves three times). The casting: with $B = 2.0 \times 10^6 \text{ s/m}^2$, a 20 mm cube ($V/A = 3.33 \text{ mm}$) freezes in $2.0 \times 10^6 \times (3.33 \times 10^{-3})^2 = \mathbf{22.2 \text{ s}}$; a 100×100×10 mm plate ($V/A = 5.00 \text{ mm}$, faces only) takes **50.0 s** – $2.25 \times$ longer *despite being thinner*, because V/A , not thickness, times the clock ✓ (the ratio is exactly $(5.00/3.33)^2 = 2.25$).

Worked example 1 – size the riser. A casting solidifies last where V/A is largest – so the riser (the reservoir that feeds shrinkage) must have a *bigger* V/A than any section it feeds. The plate's 50.0 s must be beaten: a riser of $V/A = 6.0 \text{ mm}$ gives 72.0 s ✓ and the casting is sound; a riser at $V/A = 4 \text{ mm}$ freezes first and the plate sucks in air – the rule decides, not intuition.

Worked example 2 – escalate to the certified route. The mount bracket's service load is impact (course 3), so ductility matters: the certification picks quenched + tempered 600°C (620 MPa, 16%) over the 200°C temper (950 MPa, 8%) – trading 330 MPa of strength for doubled elongation, because the drop test's joules are absorbed by plastic area, and unit 4.2's toughness is the currency.

Common mistakes. Buying to grade, not state (the menu *is* the material). Tempering too cold for an impact-loaded part (martensite's strength is not the mount's

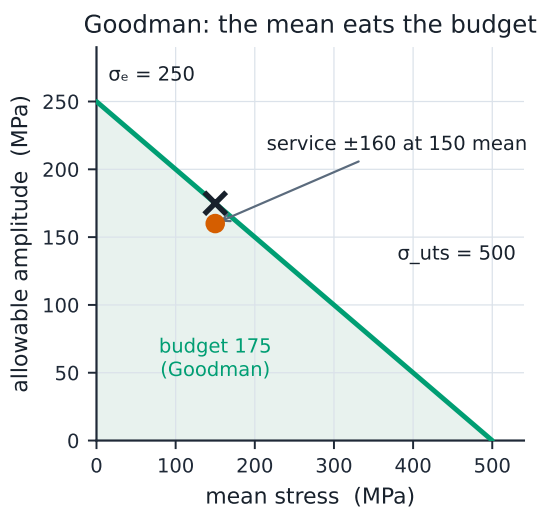
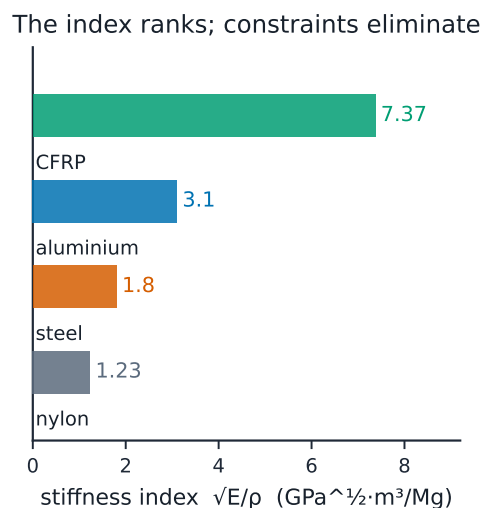
need). Comparing solidification times by thickness instead of V/A .

Summary. Four routes: 280/24, 260/26, 620/16, 950/8 – MPa vs %. Chvorinov $t = B(V/A)^2$: cube 22.2 s, plate 50.0 s, ratio $2.25 = (V/A)^2$. The riser must be greediest.

Failure and material selection

Why it matters. The mount must survive 10^7 shaker cycles, 25 festival seasons outdoors and one drop – three failure modes, one choice, and a method that shows its arithmetic.

The idea, with a picture. Fatigue's falling curve, corrosion's quiet ledger, and the Ashby index that ranks the four candidates for a stiffness-critical bracket.



*Fatigue prices the cycles; the index ranks the candidates

The definitions. **Fatigue** fails below yield under repeated load. Basquin's law $\sigma_a = \sigma'_f(2N)^b$ models a stated stress-life range. Many steels show an apparent endurance limit under specified test conditions, while typical aluminium fatigue curves keep falling. An endurance limit is not a guarantee of unlimited life under arbitrary surface, environment and loading conditions. A mean stress eats the budget – the Goodman line $\sigma_a/\sigma_e + \sigma_m/\sigma_{uts} = 1$. **Corrosion** is electrochemical loss, budgeted as thickness per year. A **material index** ranks candidates for a load case: for a stiffness-critical bracket, $\sqrt{E}/\rho$.

The derivation. Fatigue, bracket steel $\sigma'_f = 900$ MPa, $b = -0.10$: at 10^6 reversals $\sigma_a = 900 \times 10^{-0.6} = \mathbf{226.1}$ MPa; at 10^8 , $900 \times 10^{-0.8} = \mathbf{142.6}$ MPa ✓. With a 150 MPa mean ($\sigma_{uts} = 500$, $\sigma_e = 250$): Goodman allows $\sigma_a = 250(1 - 150/500) = \mathbf{175.0}$ MPa ✓ – the mean load cost 51 MPa of amplitude. Corrosion: 0.12 mm/yr $\times$ 25 yr = **3.0** mm – the wall is bought up front. The index, $\sqrt{E}/\rho$ in GPa^{1/2} · m³/Mg: steel $\sqrt{200}/7.86 = \mathbf{1.80}$, aluminium $\sqrt{70}/2.70 = \mathbf{3.10}$, nylon $\sqrt{2.0}/1.15 = \mathbf{1.23}$, CFRP

$\sqrt{139.2}/1.60 = 7.37$ ✓ – the laminate wins the stiffness-per-mass game by $2.4\times$ over aluminium.

Worked example 1 – the shaker certificate. Course 3's table cycles at 4.77 Hz between accelerations that load the bracket to ± 160 MPa about a 150 MPa mean. Goodman budget is 175 MPa: 160 fits, margin 8.6% – and Basquin extrapolates the life beyond 10^7 reversals at that amplitude ✓. The certificate reads: survives the shaker, 25 seasons – with the arithmetic attached.

Worked example 2 – escalate to the full choice. Constraints: T_g above 60°C (kills plain nylon – 55), corrosion budget 3 mm in steel vs none in CFRP, cost bracket, and the impact toughness of unit 4.2. The index says CFRP (7.37) for stiffness-per-mass; the drop test's joules and the budget say quenched-and-tempered steel. The mount ships as steel; the *carry-handle* – stiffness-critical, light, unloaded in impact – goes CFRP: one method, two right answers, both shown with their index lines.

Common mistakes. Designing fatigue to yield (fatigue lives below it); ignoring the mean stress (Goodman is not optional); comparing index values across *different* load cases (each case has its own exponent); forgetting that the index ranks – constraints eliminate.

Summary. Basquin: $226.1 \rightarrow 142.6$ MPa over $10^6 \rightarrow 10^8$; Goodman cuts 250 to 175 at 150 mean. Corrosion: 3.0 mm per 25 yr. Index $\sqrt{E}/\rho$: CFRP 7.37 > aluminium 3.10 > steel 1.80 > nylon 1.23.

The project sheet (reproducible)

The mount's material file, one page:

- Structure: BCC 0.6802, FCC 0.7405; Hall-Petch 183.6 MPa at 50 μm , 262.3 at 10 μm (gain 78.7).
- Elasticity: rod 191.0 MPa $\rightarrow$ 0.477 mm (steel) vs 1.364 mm (aluminium, ratio 200/70); true strain at 0.20 is $\ln 1.2 = 0.1823$; toughness 176 MJ/m³.
- Alloys: 0.40% C below $727^\circ\text{C} = 51.2\%$ pearlite + 48.8% ferrite; work-hardened 401.7 MPa true at 25%.
- Non-metals: laminate $E_1 = 139.2$, $E_2 = 7.36$ GPa ($18.9\times$); nylon T_g 55°C ; ceramic survival 0.983 / 0.368 / 0.002 at 200 / 300 / 360 MPa ($m = 10$).
- Processing: routes 280/24, 260/26, 620/16, 950/8; Chvorinov cube 22.2 s vs plate 50.0 s = $(V/A)^2$.
- Failure: Basquin $226.1 \rightarrow 142.6$ MPa; Goodman 175.0; corrosion 3.0 mm/25 yr; index 7.37 / 3.10 / 1.80 / 1.23 $\rightarrow$ CFRP handle, steel mount.

Recompute any line from the constants above – the steel ships quenched and tempered at 600°C , the certificate carries Goodman, Basquin and the drop-test budget side by side.