Engineering Mechanics – Statics

University Mechanical Engineering

What this course is, and how to use it

This is the first course in University Mechanical Engineering: **engineering statics** – the mechanics of bodies in equilibrium. Everything a machine, bridge or building does begins here, and the course's single central tool is the **free-body diagram**.

- Units 1.1 and 1.2 build the vocabulary: vectors, forces, and the discipline of isolating a body before computing anything.
- Units 1.3 to 1.5 solve equilibrium: particles first, then moments and supports for rigid bodies.
- Units 1.6 to 1.8 apply the same three equations to structures, friction and distributed loads.

How to read each unit: a short story says why it matters; a picture comes before any formula; then the precise definitions; then worked examples that get harder, each with a check computed a second way; then the mistakes people actually make; then a three-line summary. The habit that decides success: draw the complete free-body diagram first, with every force and every reaction, and audit it before writing equations. A missing reaction is invisible in the algebra – it simply produces a confident wrong answer.

Vectors and forces

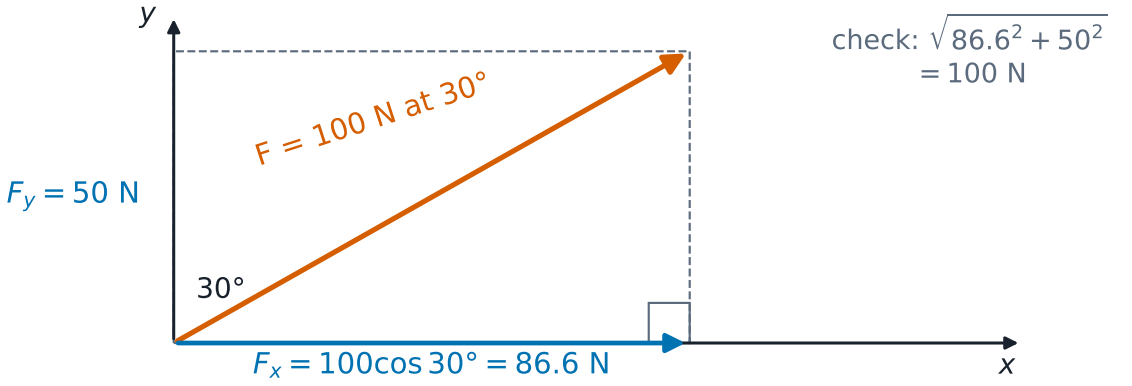
Two ropes pull a crate: one east at 30 N, one north at 40 N. Which single pull would replace them? Answering that – combining and splitting pushes – is what unit 1.1 makes precise.

A **force** is a push or pull with a **magnitude**, a **direction** and a **line of action**. Forces are vectors: they add tip-to-tail, and they resolve into components along any convenient axes. The resolution equations, with θ measured from the x-axis:

$$F_x = F\cos\theta \qquad F_y = F\sin\theta$$

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Components are the sides; the force is the diagonal



Components are the sides of the rectangle; the force is its diagonal

The **resultant** of several forces is their vector sum: add the components, then rebuild the magnitude $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$ and the direction $\theta_R = \tan^{-1}(\Sigma F_y / \Sigma F_x)$. A force can be **moved along its line of action** without changing its effect on a rigid body – but never sideways off that line. Forces in SI are measured in **newtons** (N); 1 N accelerates 1 kg at 1 m/s^2. Weights are forces: $W = mg$, with $g = 9.81$ m/s^2, so a 50 kg block weighs 490.5 N.

> **Worked example 1 (resolve and check).** A 100 N force acts at 30° above the horizontal: $F_x = 100\cos 30° = 86.6$ N and $F_y = 100\sin 30° = 50$ N. Check: $\sqrt{86.6^2 + 50^2} = 100$ N – the components always rebuild the original vector, which is a free check in both directions.

> **Worked example 2 (three forces, escalated).** A bollard is pulled by three ropes: 200 N east, 150 N north, 100 N west. Sum the components first: $\Sigma F_x = 200 - 100 = 100$ N and $\Sigma F_y = 150$ N, so $R = \sqrt{100^2 + 150^2} = 180.3$ N at $\theta_R = \tan^{-1}(150/100) = 56.3°$ above east. The check by geometry: 100 and 150 are the legs of the rectangle, and 180.3 is its diagonal – the same picture as example 1, run backwards. This components-first routine replaces every tip-to-tail drawing with arithmetic that cannot tangle.

Common mistakes. Mixing the angle's reference (from the x-axis or from the force's own line) – always state which. Using cos on both components because the triangle "looks like" a cos problem: F_x takes the cosine of the angle from the x-axis, F_y the sine, every time. Reporting a magnitude without a direction – a resultant is both.

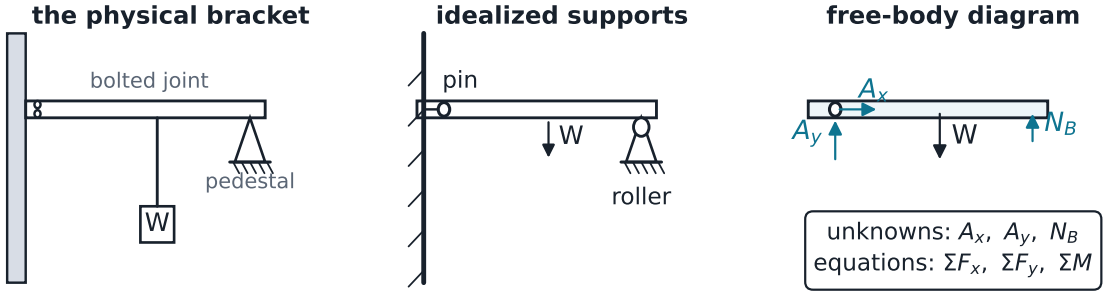
Summary: forces are vectors with magnitude, direction and line of action; resolve by cos/sin, combine by summing components, and check by rebuilding the original; move a force along its line freely, never across.

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Free-body diagrams

A book rests on a table. What does the table do to the book? Nothing in the book's picture shows it – until you cut the contact and draw what it transmits. That cut, and the discipline around it, is the free-body diagram: the course's one tool every later unit uses.

A **free-body diagram** isolates one body, draws every external force acting on it, and nothing else. It is a deliberate cut through the supports, joints and contacts, with each cut replaced by whatever it transmits.

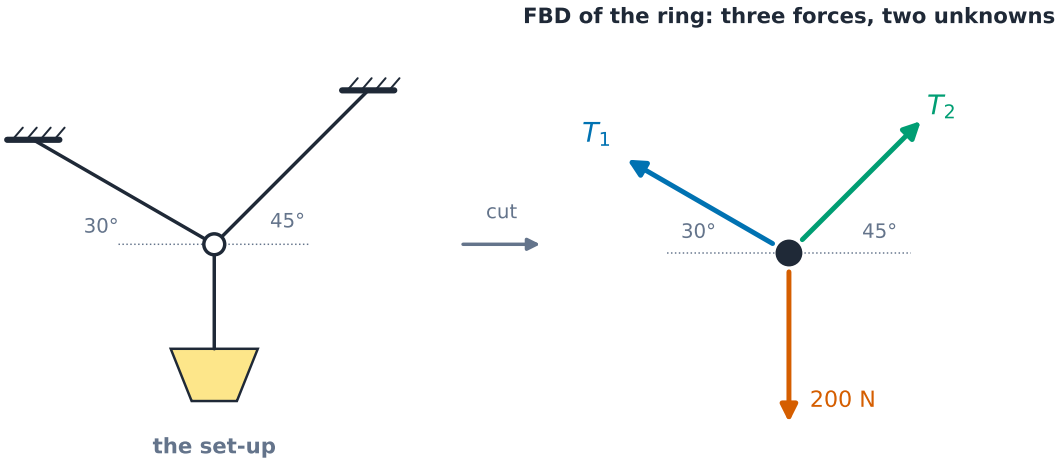


Real object, idealized setup and free-body diagram

- **Applied loads** : gravity, cables, contacts, fluid pressure – with direction and point of application.
- **Reactions** : what each support pushes back with. The weight acts at the **centre of gravity** .
- **Internal forces never appear**: if you did not cut through the member, its force is not on this diagram.

The audit: count the unknowns, count the independent equations (two for a particle in 2D, three for a rigid body), and state whether the body is **statically determinate** – solvable by equilibrium alone.

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Cut the cables and keep only what they transmit: the ring's FBD has three forces and two unknowns.

> **Worked example 1 (the lamp ring).** A lamp hangs from two cables at 30° and 45° from horizontal. The FBD of the ring is one point with three forces: the 200 N weight down and two unknown tensions along their cables. Nothing is solved yet – the diagram IS the deliverable of this step, and unit 1.3 solves it.

> **Worked example 2 (audit the beam, escalated).** A 6 m beam sits on a pin at A and a roller at B and carries one known load. The FBD shows the load, the pin's two components (A_x, A_y) and the roller's one (B_y): three unknowns against three rigid-body equations – determinate, solvable. Now audit the failure modes: miss the pin's horizontal component and the count looks friendlier but the beam is unsolved; add a second fixed end and the count rises to six against three – indeterminate, and no amount of algebra will close the gap (unit 1.5 returns to this).

Common mistakes. Drawing forces the body does not feel (the load on the *other* side of a cut). Forgetting a reaction because "the support looks rigid" – rigid is exactly why it pushes back. Drawing internal member forces on the assembly instead of cutting through them.

Summary: cut, replace, and draw only what the isolated body feels; loads have direction and point of application; the audit (unknowns vs equations) comes before any algebra.

Particle equilibrium

The lamp ring from 1.2 hangs motionless. Three forces meet there – what must be true of them? **Particle equilibrium** is the answer, and it is only two equations.

A **particle** is a body whose size does not matter – forces all act at one point.

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Equilibrium in 2D is two equations:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

Every force must be resolved first (unit 1.1); the equations then hold component by component. Choose axes along and perpendicular to the unknown you want to avoid: each equation then solves one unknown directly. When three forces balance, they must be **concurrent** – their lines of action pass through one point – or parallel; that geometric fact solves many problems without arithmetic.

> **Worked example 1 (the symmetric sign).** A 400 N sign hangs from two symmetric cables at 30° above horizontal. By symmetry each cable carries the same tension T; vertically, $2T\sin 30° = 400$, so $T = 400$ N. Horizontally the components cancel by symmetry – and if they did not, that would be the check that catches a blunder. Note how a steeper cable (larger angle) needs less tension: the vertical component does the holding.

> **Worked example 2 (the unsymmetric lamp, solved, escalated).** The 200 N lamp of unit 1.2: cable 1 at 30°, cable 2 at 45°. The equations, with T_1's x-component pointing left and T_2's right:
>
> $$T_1\cos 30° = T_2\cos 45° \qquad T_1\sin 30° + T_2\sin 45° = 200$$
>
> Substituting $T_1 = T_2\cos 45°/\cos 30° = 0.816\,T_2$ into the vertical equation: $T_2(0.408 + 0.707) = 200$, so $T_2 = 179.3$ N and $T_1 = 146.4$ N. Check: $146.4\sin 30° + 179.3\sin 45° = 73.2 + 126.8 = 200$ N exactly. The escalation to notice: symmetry died, the two-equation machinery did not – and the steeper 45° cable carries more, exactly as example 1 predicted.

Common mistakes. Putting both x-components with the same sign because the cables "both go up" – up is the y-axis; the x-components oppose. Solving by proportion (T splits like the angles) – tension follows sines, not angle sizes. Skipping the second equation when symmetry exists: symmetry is a conclusion to verify, not to assume.

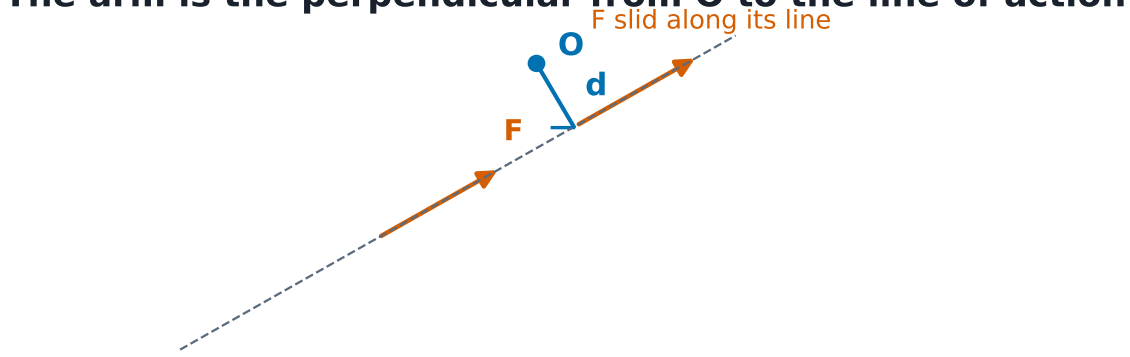
Summary: two component equations; choose axes to isolate unknowns; three forces in equilibrium are concurrent; verify symmetry instead of assuming it.

Moments and couples

Why does a longer spanner loosen a bolt that a short one cannot? The same push, a longer lever – the turning effect grows with the distance. That effect needs its own measure and its own equation.

A **moment** is the turning effect of a force about a point: $M = F \cdot d$, where d is the **moment arm** – the perpendicular distance from the point to the force's line of action. Moments are in N · m, and they carry a sign: counterclockwise positive is the usual convention.

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The moment arm is the perpendicular distance – sliding the force along its line changes nothing

A **couple** is a pair of equal, opposite, parallel forces. Its moment $M = F \cdot d$ (the distance between the lines of action) is the same about **every** point – a couple has no resultant force, only a pure turning effect. The third equilibrium equation is $\Sigma M_O = 0$ about any convenient point, and the point can be chosen on an unknown force's line to eliminate it.

Deriving why a couple is reference-free. Take moments of the pair about any point O at distance a from the first force's line: $M_O = F(a + d) - F(a) = Fd$ – the a cancels. One force gains, the other loses, by exactly the same amount. That cancellation is the definition of "pure turning".

> **Worked example 1 (couple versus single force).** Two 150 N forces form a couple with lines of action 0.8 m apart: $M = 150 \times 0.8 = 120$ N · m, the same about every point in the plane. A single 240 N force with a 0.5 m moment arm also gives 120 N · m about *that* point – but only about that point; move the reference point and the moment changes. That contrast – couple constant, single-force moment reference-dependent – is the whole idea.

> **Worked example 2 (combine them, escalated).** A bracket arm feels a downward 300 N force whose line of action passes 0.4 m to the right of bolt O, plus an applied clockwise couple of 60 N · m. About O: the force gives $300 \times 0.4 = 120$ N · m clockwise, the couple 60 N · m clockwise, total 180 N · m clockwise – and about any *other* point the force part changes while the 60 N · m never does. If the bracket is in equilibrium, the bolt and its mate must supply 180 N · m of counterclockwise moment between them; the reference-independence lets you take moments about whichever bolt suits the algebra.

Common mistakes. Using the distance to the force's *point of application* instead of the perpendicular distance to its line of action. Dropping the couple from moment

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sums “because it has no force” – a couple enters EVERY moment equation, fully. Forgetting moments are signed: mixing clockwise and counterclockwise magnitudes in one sum.

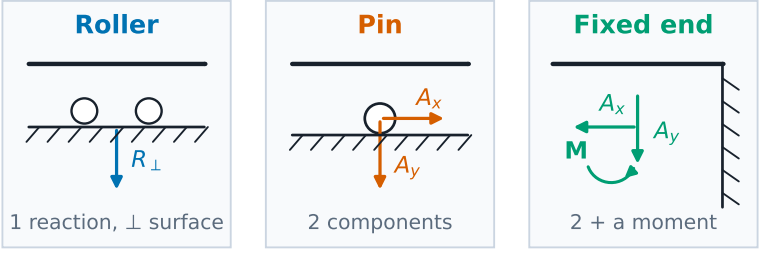
Summary: $M = Fd$ about a chosen point with the perpendicular arm; couples are pure turning, the same about every point; choose the moment point to cancel unknowns.

Rigid-body equilibrium and supports

A bridge girder rests on one bolted end and one sliding end – why one bolt and not two? Because the supports' job is to supply exactly the unknowns the three equations can consume: this unit assembles forces and moments into the full rigid-body method.

A rigid body in 2D gives three equations: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$. Supports supply reactions, and each support type constrains a specific motion:

One, two, or two-plus-a-moment: supports priced by what they prevent

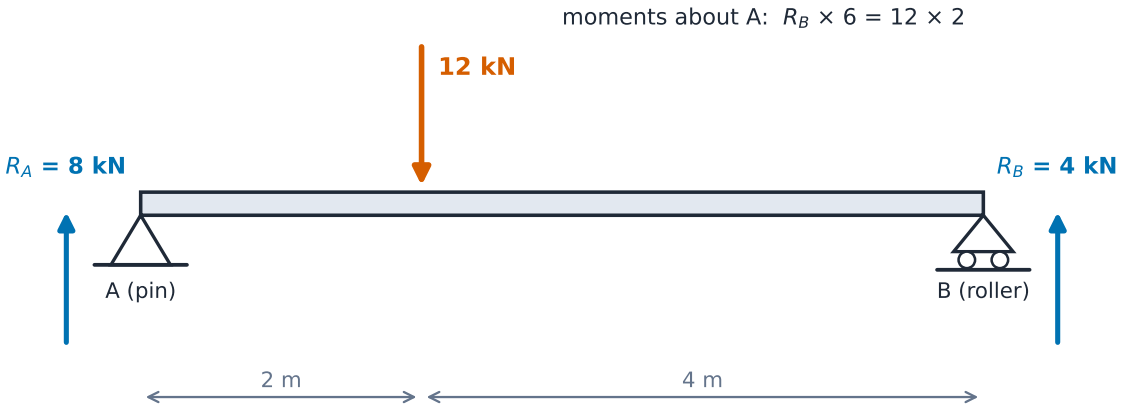


Roller: one reaction; pin: two; fixed end: two plus a moment

- **Roller** : one reaction, perpendicular to the surface.
- **Pin** : two reaction components, any direction.
- **Fixed support** : two force components **and** a moment.

> **Worked example 1 (the simple beam).** A 6 m beam on a pin at A and a roller at B carries a 12 kN point load 2 m from A. Moments about A: $R_B \times 6 = 12 \times 2$, so $R_B = 4$ kN; vertically $R_A = 12 - 4 = 8$ kN. Check by moments about B: $R_A \times 6 = 8 \times 6 = 48 = 12 \times 4$ ✓ – an equation that was not used to produce either answer.

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Moments about A give R_B = 4 kN; the vertical sum gives R_A = 8 kN.

Worked example 2 (the overhang, escalated). The same beam, but the roller moves to 4 m from A and the 10 kN load hangs at the 6 m tip. Moments about A: $R_B \times 4 = 10 \times 6$, so $R_B = 15$ kN – larger than the load, because it holds a lever arm advantage. Vertically $R_A = 10 - 15 = -5$ kN: the pin pulls *down* by 5 kN. Check by moments about B: $R_A \times 4 + 10 \times 2 = -20 + 20 = 0$ ✓. The negative answer is not an error – it is the beam telling you the hold-down must exist, and it is why real overhangs are bolted, not just rested.

A **two-force member** carries equal, opposite forces along the line joining its two attachment points – no other loading. Spotting them turns hard structures into easy ones. Every check here recomputes an answer through an equation that was not used to produce it.

Common mistakes. Assuming every vertical reaction points up – overhangs and offset loads flip them. Taking moments about several points but using the same unknown's wrong lever arm twice: draw the distances on the FBD before summing. Calling an indeterminate assembly "solvable with more effort": 6 unknowns against 3 equations is a fact, not a challenge.

Summary: three equations, supports priced by what they prevent (1, 2, or 2+M unknowns); moment about the point that kills an unknown; negative reactions are information; two-force members are gifts.

Structures – trusses and frames

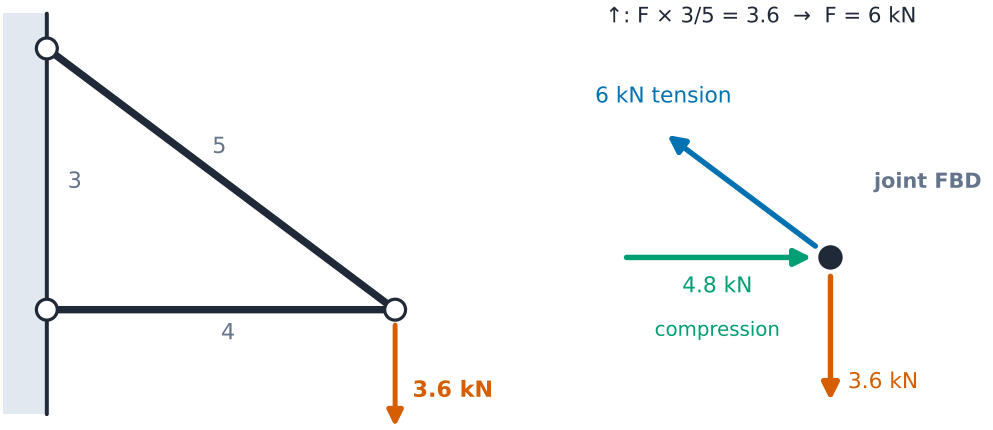
A railway bridge carries its train through hundreds of slim bars, none of which bends. The trick is the triangle: pinned triangles carry load axially, and axial means two-force means solvable – this unit is the payoff of everything before it.

A **truss** is a structure of straight two-force members joined at frictionless pins, loaded only at the joints. Members carry pure **tension** or **compression**

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along their axes.

Method of joints : at each joint, two force equations solve two unknown member forces. **Method of sections** : cut through three members, take one piece as a FBD, and solve three equations – ideal for one member far inside a long truss. **Zero-force members** carry zero axial force under the specified loading. At a joint with no external load or support reaction, two non-collinear members are both zero-force members. If three members meet and two are collinear, the third is a zero-force member. Symmetry is not required.



The joint's two equations: 6 kN tension in the inclined member, 4.8 kN compression in the horizontal one.

Worked example 1 (a joint at the wall). A joint at the wall is held by a horizontal member and an inclined member rising at 3-4-5 geometry, carrying a 3.6 kN load hanging down. Vertically: $F_{inclined} \times (3/5) = 3.6$, so $F_{inclined} = 6$ kN (tension). Horizontally the horizontal member takes $6 \times (4/5) = 4.8$ kN compression. Every joint of every truss is this same two-equation step.

Worked example 2 (a full truss, machine-verified, escalated). A Pratt truss of 6 m span in three 2 m panels, 2 m tall: bottom joints A–C–D–B, top joints T1 above C and T2 above D, verticals at C and D, diagonals A–T1 and T2–B, and one interior diagonal T1–D. Loads of 4 kN hang at C and D; by symmetry $R_A = R_B = 4$ kN. Solving all six joints (machine-verified, every residual zero):

- bottom chords: 4 kN tension in every panel; top chord T1–T2: 4 kN compression
- end diagonals A–T1 and T2–B: $4\sqrt{2} = 5.66$ kN compression; verticals: 4 kN tension (each one hangs its bottom joint's load from the top chord)
- the interior diagonal T1–D: **zero** – the middle panel carries no shear, because the 4 kN reaction is used up by the 4 kN load at C

This is the pattern of every simply supported truss under downward loads: it sags,

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so the bottom chord stretches (tension) and the top chord shortens (compression). If your bottom chord comes out in compression, look for a sign slip.

The section method cross-checks one member without the joints: cut the right panel (bottom chord D–B and diagonal T2–B), keep the right piece (joint B) and take moments about T2, where the cut diagonal passes. With tension positive, the bottom-chord force pulls B to the left along a line 2 m below T2: $4 \times 2 - F_{bottom} \times 2 = 0$, so $F_{bottom} = 4$ kN tension – agreeing with the joint-by-joint result through a different route.

Frames and machines have multi-force members; dismember them, draw each member's FBD with the joint forces equal and opposite on the neighbour, and solve the assembly piece by piece.

Common mistakes. Loading a joint midway along a member – truss members are loaded at joints only (midway loading bends the member out of the theory). Assuming every diagonal "must" carry load: find the zero-force members first. Applying sections through four members: three cut members is the maximum three equations can buy.

Summary: joints give two equations each, sections three; zero-force members shrink the work; compression vs tension is the report; frames dismember into equal-and-opposite pairs.

Friction

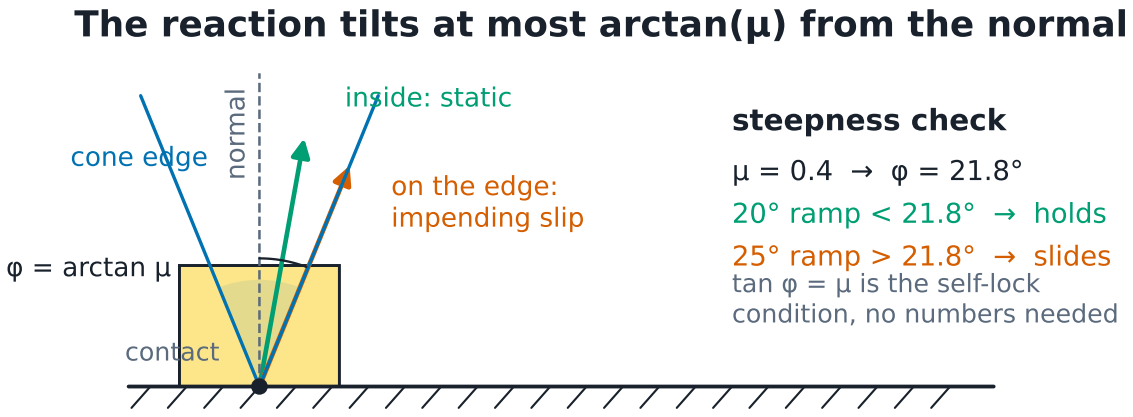
A 50 kg crate sits on a 20° ramp and does not slide. Why not – and how steep would be too steep? Friction is the one force in the course that adjusts itself, and the whole unit is learning to ask "how much does it *need*?" before computing anything.

Dry **friction** opposes relative sliding at a contact. Before sliding, the friction force adjusts itself to whatever maintains equilibrium, up to a limit proportional to the normal force:

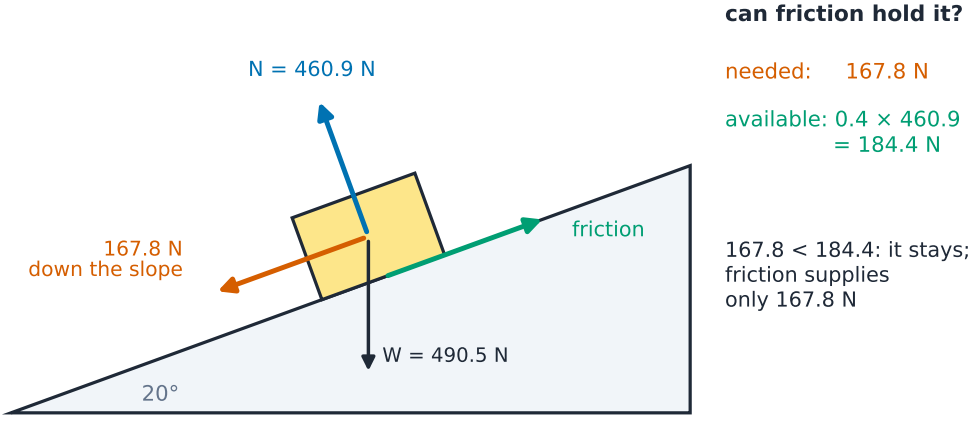
$$F \le \mu_s N \qquad F_{kinetic} = \mu_k N$$

Here the **coefficient of static friction** sets the limiting static friction, while the **coefficient of kinetic friction** applies during sliding. At the limit the contact is in **impending slip** and the full friction cone applies: the total reaction $R = \sqrt{N^2 + F^2}$ tilts at most $\phi = \tan^{-1} \mu_s$ from the normal.

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The reaction tilts at most arctan(μs) from the normal – on the edge at impending slip



Friction needs 167.8 N of the 184.4 N available, so the block stays put.

Worked example 1 (the incline, and the classic comparison). A 50 kg block on a 20° incline ($\mu_s = 0.4$). Weight component down the slope: $490.5 \sin 20° = 167.8$ N; available friction: $0.4 \times 490.5 \cos 20° = 184.4$ N. Because $167.8 < 184.4$, the block stays – and the friction force actually acting is only 167.8 N, not 184.4 N. Friction takes what it needs, never what it could supply. The steepest no-slip angle follows at once: $\tan^{-1}(0.4) = 21.8°$, just above 20° – the margin is real but thin.

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Worked example 2 (tip or slide, escalated). A crate 0.5 m wide and 1.0 m tall, mass 40 kg ($W = 392.4$ N), $\mu_s = 0.5$, is pulled horizontally at its top edge. Sliding needs $P_{slip} = \mu_s W = 0.5 \times 392.4 = 196.2$ N. Tipping about the front bottom edge balances the pull against the weight's moment: $P_{tip} \times 1.0 = 392.4 \times 0.25$, so $P_{tip} = 98.1$ N. The smaller one happens first: the crate **tips** at 98.1 N, well before any sliding. The design lesson: tall crates on grippy floors tip; low crates on slippery floors slide – pull low and the same crate slides instead.

Common mistakes. Setting $F = \mu N$ always: that is the *limit*; equilibrium sets the actual force, and only impending slip reaches the limit. Getting the would-be motion's direction wrong – friction opposes the motion that *would* occur, not any motion you can imagine. Comparing P values without checking which failure they belong to.

Summary: friction self-adjusts up to $\mu_s N$; compare needed against available; the cone angle $\tan^{-1}\mu_s$ is the steepest self-locking tilt; tip-versus-slide compares two pulls and takes the smaller.

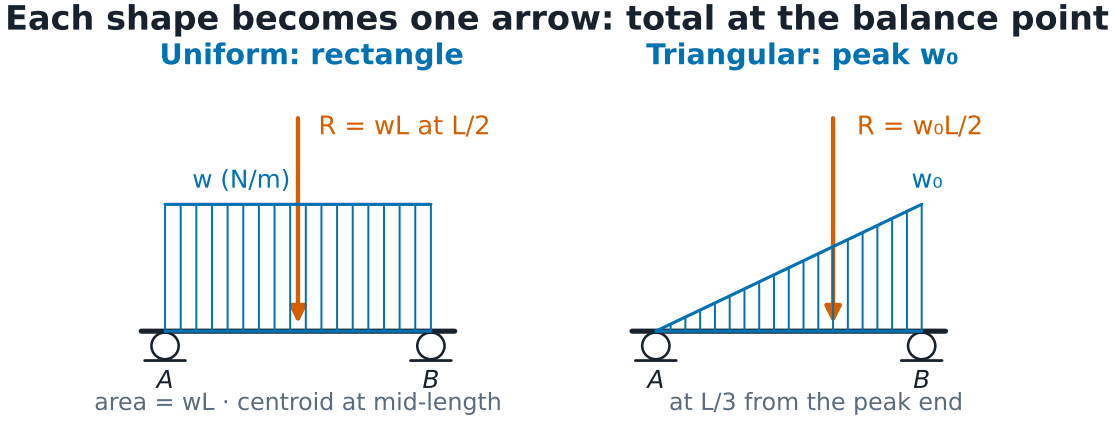
Centroids and distributed loads

Snow piles deeper at mid-span than at the eaves – the beam below does not feel a point, it feels a shape. Engineering answers with one move: replace the shape by its total at its balance point, and every earlier tool applies unchanged.

A **distributed load** spreads force over a length or area (N/m, N/m^2). A **centroid** is the load's balance point: the whole distribution can be replaced by its resultant – total magnitude acting at the centroid.

- Rectangle (uniform w over length L): $R = wL$ at the midpoint, $L/2$.
- Triangle (peak w_0 at one end over L): $R = w_0 L/2$ at $L/3$ from the peak end.

$$\Sigma M: \text{ moment of the resultant} = \text{moment of the distribution}$$



Each shape becomes one arrow: total at the balance point

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Deriving the triangle's centroid. A strip at x from the zero end carries intensity proportional to x, so $\bar{x} = \int x \cdot (w_0 x/L)\,dx \,/\, \int (w_0 x/L)\,dx = 2L/3$ from the zero end, which is $L/3$ from the peak – the one-third rule, now with its origin visible.

> **Worked example 1 (the triangular beam).** A triangular load rises from zero to 6 kN/m over 4 m on a simply supported beam. Total: $R = 6 \times 4/2 = 12$ kN acting at $4/3 = 1.333$ m from the high end. Reactions follow from two moment equations exactly as for a point load. When a uniform and a triangular load share a beam, replace each by its resultant and sum – the replacement is exact, not an approximation.

> **Worked example 2 (two shapes, machine-verified, escalated).** A 6 m beam carries a uniform 2 kN/m over the first 3 m and a triangle rising to 4 kN/m over the last 3 m. Resultants: $R_1 = 6$ kN at 1.5 m from A; $R_2 = 6$ kN at 5 m from A. Moments about A: $R_B \times 6 = 6 \times 1.5 + 6 \times 5 = 39$, so $R_B = 6.5$ kN and $R_A = 5.5$ kN. Check about B: $5.5 \times 6 = 33 = 27 + 6$ ✓ (machine-verified).

With this the course closes its loop: loads reduce to resultants, resultants enter free-body diagrams, and the three equilibrium equations finish the job.

Common mistakes. Placing the triangular resultant at the geometric middle – it sits $L/3$ from the peak end, always. Splitting a trapezoid with the wrong peak ends. Treating the replacement as approximate: it is exact.

Summary: each shape becomes its total at its centroid – wL at $L/2$, $w_0 L/2$ at $L/3$ from the peak – then unit 1.5 finishes, exactly.

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