Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

distance = area = $\tfrac{1}{2}(4)(12) + 6(12) + \tfrac{1}{2}(3)(12) =$ **114 m**; steepest slope $\dfrac{12}{3} =$ **4.0 m/s²** (braking)

2.2

$\vec{v} = 6t\,\hat{\imath} + (6t^2 - 1)\,\hat{\jmath} = (12,\ 23)$ m/s, $|\vec{v}| =$ **25.9 m/s**

$\vec{a} = 6\,\hat{\imath} + 12t\,\hat{\jmath} = (6,\ 24)$ m/s², $|\vec{a}| =$ **24.7 m/s²**

2.3

$\displaystyle\int_2^0 v\,dv = \int_0^x -25x\,dx$; $-2 = -12.5x^2$; $x =$ **0.40 m**

2.4

range $= \dfrac{v^2 \sin 2\theta}{g} = \dfrac{225 \sin 80^\circ}{9.81} =$ **22.6 m**; height $= \dfrac{(15 \sin 40^\circ)^2}{2g} =$ **4.74 m**

3.1

$v = \displaystyle\int_0^t a\,dt = 1.2\left(t - \frac{t^2}{80}\right)$, $v(40) =$ **24 m/s**; check: area of the triangle $\tfrac{1}{2} \times 40 \times 1.2 = 24$

$x = 1.2\left(\dfrac{t^2}{2} - \dfrac{t^3}{240}\right)$, $x(40) =$ **640 m**

3.2

$r = 0.90$ m, $\dot{r} = 0.20$ m/s, $\ddot{r} = 0$, $\dot{\theta} = 0.60$ rad/s, $\ddot{\theta} = 0$; $v_r =$ **0.20 m/s**, $v_\theta = r\dot{\theta} =$ **0.54 m/s**

$a_r = \ddot{r} - r\dot{\theta}^2 =$ **−0.324 m/s²** (inwards); $a_\theta = 2\dot{r}\dot{\theta} =$ **0.24 m/s²**

$|\vec{a}| =$ **0.403 m/s²**; without sliding the Coriolis term $2\dot{r}\dot{\theta}$ is zero

3.3

(a) $v_2^2 = v_1^2 + 2a_t s$: $a_t = \dfrac{225 - 625}{160} =$ **−2.5 m/s²**

(b) $a_n = \dfrac{25^2}{60} = 10.4$ m/s²; $|\vec{a}| = \sqrt{10.4^2 + 2.5^2} =$ **10.7 m/s²**

(c) limit $0.80 \times 9.81 = 7.85$ m/s² < 10.7: **the car skids**; $a_n \le \sqrt{7.85^2 - 2.5^2} = 7.44$ m/s²; $v \le \sqrt{7.44 \times 60} =$ **21.1 m/s**, about 76 km/h, so the driver must slow down before the curve, not on it

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