2.5 Vector calculus

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS gradient • divergence theorem

Objective

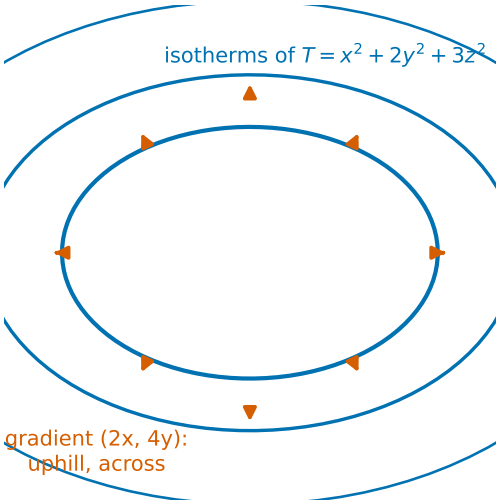
Compute grad/div/curl at a point, and use the theorems to count flux and circulation the cheap way.

1 Key knowledge and worked examples

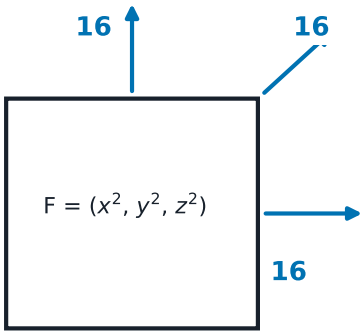
■ Operators and theorems

∇T uphill; $\nabla \cdot \mathbf{F}$ sources; $\nabla \times \mathbf{G}$ spin. Divergence theorem: closed-surface flux = volume integral; Stokes: loop circulation = curl flux.

Fields: T rises at 7.483 K/m; q = -40 grad T



Flux 48, counted two ways



surface flux: 16 + 16 + 16 = 48

divergence: 2(x+y+z) dV = 48

Closed surfaces only

■ Worked example. The amplifier

$\nabla T(1,1,1) = (2,4,6)$, $|\nabla T| = 7.483$; $\mathbf{q} = -40\nabla T = (-80,-160,-240)$ W/m2; cube flux $48 = 48$ ✓.

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2 Practice

A heater's field is $T = 3x^2 + y^2 + 2z^2$ (k = 40 W/mK), and a flow is $\mathbf{G} = (0, 0, xy)$.

2.1 ∇T at (1, 1, 1), its magnitude, and the heat flux vector with its magnitude (one decimal). [4]

2.2 The flux of $\mathbf{F} = (x^2, y^2, z^2)$ through the cube $[1, 3]^3$: by faces, and by the divergence theorem. [4]

2.3 $\nabla \times \mathbf{G}$ at (1, 1, 2). [3]

2.4 The grille has one panel missing. State the theorem's status and the standard repair in one line each. [3]

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