Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\det = 0.36 + 0.64 = 1$; $\vec{F} = \begin{pmatrix} 0.6 & 0.8 \\ -0.8 & 0.6 \end{pmatrix} \begin{pmatrix} 0 \\ 10 \end{pmatrix} = (\mathbf{8.0},\ \mathbf{6.0})$ **kN**

2.2

$R_2 - 2R_1$: $-3y - z = -9$; $R_3 - R_1$: $y - 2z = -4$

from these: $y = 2$, $z = 3$; then $x = 6 - 2 - 3 = \mathbf{1}$: $(x, y, z) = (\mathbf{1}, \mathbf{2}, \mathbf{3})$

2.3

$0.001y = 0.002$, so $y = 2$, $x = \mathbf{0}$: the answer is $(0,\ 2)$

a 0.05% change in the data moved x from 1 to 0; $\det A = 0.001$ is tiny: the lines are almost parallel, the system is **ill-conditioned**

2.4

$\lambda^2 - 20\lambda - 2400 = 0$; $\lambda = 10 \pm 50$: **60 MPa** and $-$**40 MPa**; $\tan 2\theta_p = 0.75$, $\theta_p = \mathbf{18.4}°$

3.1

$\dfrac{K}{m} = \begin{pmatrix} 800 & -400 \\ -400 & 400 \end{pmatrix}$; $\lambda^2 - 1200\lambda + 160\,000 = 0$

$\omega^2 = 152.8$ and 1047 rad^2/s^2; $f = \dfrac{\omega}{2\pi} =$ **1.97 Hz** and **5.15 Hz**

mode 1: $x_2/x_1 =$ **1.62**: both carts move the same way, cart 2 further; mode 2: $x_2/x_1 =$ $-$**0.62**: they move in opposite directions

3.2

$a + b = 3$, $a - b = 4$: $a =$ **3.5**, $b =$ $-$**0.5**

$\hat{e}_1$, $\hat{e}_2$ are orthonormal (perpendicular, length 1), so each coefficient is a dot product: $\vec{v} \cdot \hat{e}_1 = 7/\sqrt{2}$, with no equations to solve

3.3

(a) $\det = 2\sin\theta\cos\theta = \sin 2\theta = 0.866$; $F_1 = F_2 = \dfrac{P}{2\sin\theta} =$ **10 kN**

(b) $F = \dfrac{10}{2\sin 2.0°} =$ **143 kN**; $\det = \sin 4° =$ **0.070**

(c) $F = \dfrac{10}{2\sin 2.1°} = 136$ kN, a change of **4.8**% for a 5% angle error; the system is ill-conditioned when θ is small, so keep θ well away from 0 (for example above 15°)

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