UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

12.5 Digital implementation

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS zero-order hold • Nyquist frequency • aliasing • ADC

Objective

Count the phase a zero-order hold and a computation delay cost, choose a sample rate, spot aliasing, find encoder resolution, and run a Tustin PI update by hand.

1 Key knowledge and worked examples

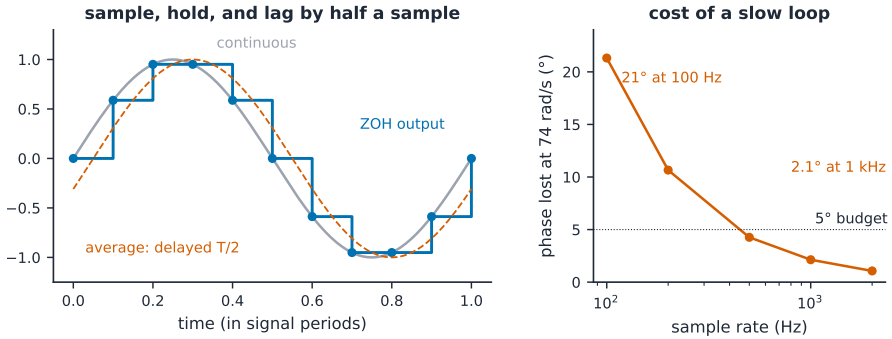
What sampling costs

- An **ADC** samples every T; the **zero-order hold** acts like a delay of $T/2$: phase lag $\omega T/2$ (rad).
- A computation delay τ_d costs $\omega\tau_d$ more.
- Above the **Nyquist frequency** $f_s/2$, a signal at f appears at $|f - kf_s|$: **aliasing**. Filter before the ADC.
- Tustin integral: $I_k = I_{k-1} + \frac{T}{2}(e_k + e_{k-1})$; PI output $u_k = K_p e_k + K_i I_k$. Sample 20 to 30 times the bandwidth.

Worked example. Choosing the sample rate

Valve loop, $\omega_n = 74.4$ rad/s, crossover 48.2 rad/s, $PM = 65.2°$.

- Hold lag at ω_n: $74.4T/2 =$ **2.1**° at 1 kHz, **21**° at 100 Hz.
- Phase margin at crossover: 63.8° at 1 kHz, 51.3° at 100 Hz.
- Encoder: 2000 counts per motor turn × 50 = 100 000 per valve turn, **0.0036**° per count.



2 Practice

The valve loop of sheet 12.3: $\omega_c = 29.8$ rad/s, $PM = 65.2°$.

2.1 Find the hold's phase lag at crossover (two decimals, degrees) and the remaining phase margin when sampling at 500 Hz and at 100 Hz. [4]

2.2 Sampling at 1 kHz, where does noise at 1.2 kHz appear? And noise at 700 Hz? [2]

2.3 A 1000-line encoder (quadrature ×4) on the motor drives the valve through 30 : 1. Find the counts per valve turn and the angle per count (degrees). [2]

3 Exam-style questions

3.1 (a) Find the motor-speed resolution from counting that encoder at 1 kHz (three decimals, rad/s) and say what it implies for derivative action. (b) A 0.5 ms computation delay is added: find its lag at crossover (two decimals, degrees). (c) With $T = 2$ ms, $K_p = 79.4$, $K_i = 365.2$, $I_0 = 0$ and errors $e_0 = 0.10$, $e_1 = 0.08$, $e_2 = 0.06$ rad, find I_1, I_2 and u_2 (three decimals, V). [8]