# 11.6 Multi-degree-of-freedom systems

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

**KEY WORDS** eigenvalue problem • mode shape • mode superposition

## Objective

Write the mass and stiffness matrices of a two-mass system, find its natural frequencies and mode shapes, and find its forced response and floor force at the running speed.

## 1 Key knowledge and worked examples

■ **Matrices, modes, superposition**

- $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}$; for a machine $m_1$ on mounts $k_1$ above a frame $m_2$ on feet $k_2$: $\mathbf{K} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix}$.
- **Eigenvalue problem** $\det(\mathbf{K} - \omega^2\mathbf{M}) = 0$ gives the natural frequencies; each **mode shape** from row 1: $u_2/u_1 = 1 - \omega^2 m_1/k_1$.
- Modes are orthogonal, so the response is a sum of single-degree responses: **mode superposition** .
- Forced: solve $(\mathbf{K} - \omega^2\mathbf{M})\mathbf{X} = \mathbf{F}_0$; floor force $k_2 X_2$.

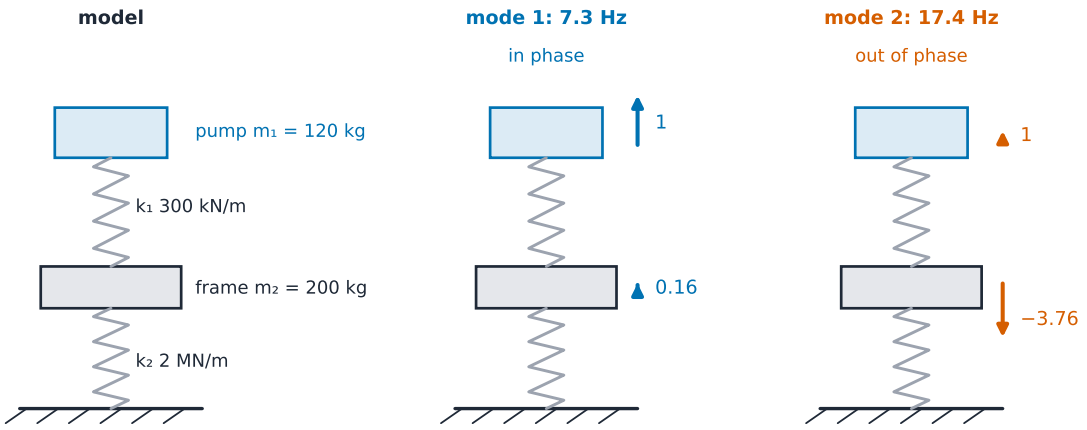
■ **Worked example. The pump on its frame**

$m_1 = 120$ kg, $k_1 = 300$ kN/m, $m_2 = 200$ kg, $k_2 = 2$ MN/m.

- $\det(\mathbf{K} - \omega^2\mathbf{M}) = 0$: $\omega_1 = \mathbf{45.8}$ rad/s (7.30 Hz), $\mathbf{u}_1 = (1, 0.160)$, in phase.
- $\omega_2 = \mathbf{109.1}$ rad/s (17.4 Hz), $\mathbf{u}_2 = (1, -3.76)$, frame against pump.
- At 24 Hz with 27.3 N on the pump: pump 11.4 µm, frame 1.5 µm, floor force $k_2 X_2 = \mathbf{3.0}$ N.



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## 2 Practice

A 100 kg fan sits on mounts of total stiffness $k_1 = 200$ kN/m on a 150 kg frame whose feet have $k_2 = 1.5$ MN/m.

**2.1** Write $\mathbf{M}$ and $\mathbf{K}$. [2]

**2.2** Show that $\det(\mathbf{K} - \omega^2\mathbf{M}) = 0$ becomes $15\,000\,\omega^4 - 2.0 \times 10^8\,\omega^2 + 3.0 \times 10^{11} = 0$, and find both natural frequencies (two decimals, rad/s and Hz). [5]



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## 3 Exam-style questions

**3.1** (a) Find both mode shapes as $(1, u_2)$ (three decimals) and describe each. (b) The fan's unbalance gives $F_0 = 49.3$ N on $m_1$ at 157.1 rad/s. Find the pump and frame amplitudes (one decimal and two decimals, µm) from $(\mathbf{K} - \omega^2\mathbf{M})\mathbf{X} = (F_0, 0)$, and the floor force (one decimal, N). (c) Compare with treating the fan alone on $k_1$ above a rigid floor, and say what would happen if mode 2 moved to 25 Hz. [10]



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