# 11.2 Velocity and acceleration analysis

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

**KEY WORDS** velocity polygon • acceleration polygon • instant centre • Coriolis component

## Objective

Find the velocities and accelerations of a slider-crank from its polygons, the angular velocity of the rod, and the Coriolis component of a block sliding on a turning arm.

## 1 Key knowledge and worked examples

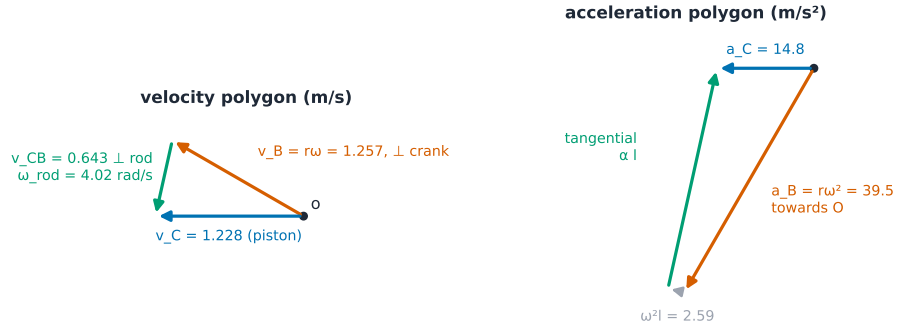
### ■ Polygons and the Coriolis term

- On one rigid link, $\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{CB}$: the relative velocity is across the link (**velocity polygon** ).
- Acceleration adds $\omega^2 r$ along the link and $\alpha r$ across it (**acceleration polygon** ).
- Slider-crank results: $v_C = r\omega \sin(\theta+\phi)/\cos\phi$, $\omega_3 = r\omega\cos\theta/(l\cos\phi)$, and $a_C \approx r\omega^2(\cos\theta + \cos 2\theta/n)$, $n = l/r$.
- **Coriolis component** $2\omega v_{rel}$ for sliding on a rotating link; an **instant centre** gives a check.

### ■ Worked example. Velocities at 60°

$r = 40$ mm, $l = 160$ mm, 300 rpm, $\theta = 60°$, $\phi = 12.5°$.

- $\omega = 2\pi N/60 = 2\pi \times 300/60 = \mathbf{31.42}$ rad/s; $v_B = r\omega = \mathbf{1.257}$ m/s.
- Polygon: $v_C = \mathbf{1.228}$ m/s, $v_{CB} = 0.643$ m/s, so $\omega_3 = v_{CB}/l = \mathbf{4.02}$ rad/s.
- $a_B = r\omega^2 = 39.5$ m/s$^2$; closing the acceleration polygon gives $a_C = \mathbf{14.8}$ m/s$^2$.




## 2 Practice

A slider-crank has $r = 50$ mm and $l = 200$ mm and turns at 600 rpm.

**2.1** Find $\omega$ (two decimals, rad/s) and the crank-pin speed $v_B$ (three decimals, m/s). [2]

**2.2** At $\theta = 45°$ ($\phi = 10.18°$) find the piston speed $v_C$ (three decimals, m/s), the rod's angular velocity $\omega_3$ (two decimals, rad/s) and $v_{CB}$. [5]

**2.3** A block slides outwards at 0.4 m/s along a slotted arm turning at 8 rad/s. Find the Coriolis acceleration and its direction. [3]

## 3 Exam-style questions

**3.1** For the slider-crank of 2.2 at $\theta = 45°$: (a) find the crank-pin acceleration and the rod's centripetal acceleration (one decimal, m/s$^2$); (b) estimate the piston acceleration with the approximate formula (one decimal, m/s$^2$) and give its direction; (c) find the inertia force on a 1.2 kg piston (nearest N) and say why the approximation is good here. [7]