1.8 Centroids and distributed loads

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS distributed load • resultant • centroid

Objective

Replace distributed loads by their resultants at the correct locations, locate centroids of composite areas, and close the course's loop: distributions → resultants → free-body diagrams → the three equilibrium equations.

1 Key knowledge and worked examples

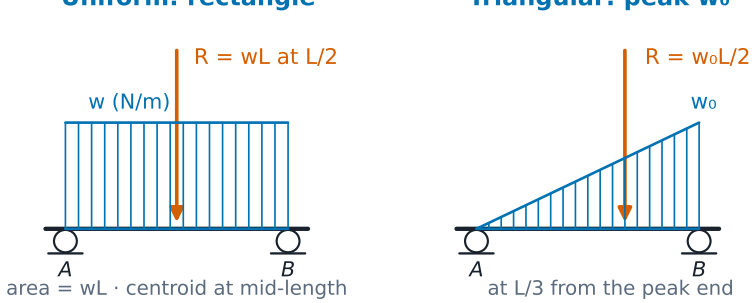
■ A distributed load is one resultant in disguise

A **distributed load** (N/m) acts everywhere along its length, but equilibrium only ever sees its **resultant** : total magnitude acting at the **centroid** of the distribution.

- Rectangle (uniform `w` over `L`): `R = wL` at `L/2`.
- Triangle (peak w_0 at one end, zero at the other): $R = w_0 L/2$ at `L/3` from the peak end.

The replacement is exact, not an approximation.

Each shape becomes one arrow: total at the balance point



■ Worked example: the triangular load

A load rising from zero to 6 kN/m over 4 m: $R = 6 \times 4/2 = 12kN$ acting at `4/3 = 1.333 m` from the high end. On a beam supported at both ends, the two moment equations then behave exactly as for a point load.

Two loads share a beam? Replace each by its own resultant and add the moments – the linearity of equilibrium does the rest.

■ Centroids by composite areas

A composite area's centroid:

$$\bar{y} = \frac{\sum A_i \, \bar{y}_i}{\sum A_i}$$

Cut the shape into rectangles, take moments of area about any convenient axis, and divide by the total area. The check: the centroid must lie inside the shape's convex hull, on the side of the bigger area.

2 Practice

2.1 A uniform 3 kN/m load covers a 5 m span. Give the resultant and its position. [2]

2.2 A triangular load peaks at 4 kN/m and falls to zero over 6 m. Give the resultant and its distance from the peak end. [3]

2.3 A 6 m simply supported beam carries a uniform 2 kN/m over its full length plus a triangular load rising from 0 to 3 kN/m over the same span (peak at the right end). Find both reactions, taking moments about the left support, and verify about the right support. [6]

3 Exam-style questions

3.1 A T-section has a flange 120 mm wide × 20 mm deep on top of a web 20 mm wide × 80 mm deep, both measured from the top of the flange.

Divide the section into two rectangles and compute the centroid height from the top. State the check you would run on the answer before trusting it. [6]

3.2 An 8 m simply supported beam (pin at A, roller at B) carries a uniform 2.5 kN/m over its full length and a 20 kN point load 3 m from A.

Replace the distributed load by its resultant, then find both reactions from moments and the vertical equation, and verify by moments about B. State why the replacement is exact rather than approximate. [6]