1.3 Particle equilibrium

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS particle • concurrent • equilibrium • free-body diagram

Objective

Solve particle equilibrium with the two component equations, choose axes that decouple the unknowns, and use the three-force concurrency fact to solve problems without arithmetic. Verify every answer through the unused equation.

1 Key knowledge and worked examples

■ Two equations, however many cables

A **particle** is a body whose size does not matter – every force acts at one point. Equilibrium is two equations:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

Draw all forces on the free-body diagram first; the equations are then bookkeeping. The unknowns can be two tensions, a tension and an angle, or a force and a reaction.

■ Worked example: symmetric cables

A 400 N sign hangs from two cables at 30° above horizontal, symmetric. By symmetry the tensions are equal; vertically:

$$2T\sin 30^\circ = 400 \;\Rightarrow\; T = 400 \text{ N}$$

Horizontally, `T cos 30°` pulls left and right equally – nothing to compute, and the 30° geometry makes the tension equal the weight. Steeper cables need less tension (60° would give $T = 231N$); flatter cables need more (10° would give `1154 N`).

■ Choose axes, then use the spare equation

Tilt the axes along and perpendicular to an unknown force: that unknown vanishes from the perpendicular equation, which then solves the other unknown directly.

Worked example. A crate on a smooth 25° incline, held by a rope along the slope. Axes along/perpendicular to the slope: perpendicular gives $N = Wcos25^\circ$ with the rope absent from the equation; along-slope gives $T = Wsin25^\circ$ for equilibrium. The unused horizontal/vertical pair then serves as the check.

Three concurrent forces: if three forces balance, their lines of action pass through one point (or are parallel). That geometric fact often names the direction of an unknown force before any algebra.

2 Practice

2.1 A 300 N weight hangs from two symmetric cables at 40° above horizontal. Find the tension in each cable, and the size of the force each cable's horizontal component must cancel. [3]

2.2 A ring is pulled by three ropes in the plane: 80 N east, 60 N north, and a third rope pulling west. Equilibrium holds. Find the third rope's force, and why no north-south force is needed. [3]

2.3 A 500 N crate rests on a smooth (frictionless) 20° incline, held by a rope parallel to the slope. Using axes along and perpendicular to the slope, find the normal force and the rope tension. [4]

3 Exam-style questions

3.1 A 200 kg crate hangs from a single pulley system: one rope segment rises vertually to the ceiling, the other leaves the pulley at 35° above horizontal and is pulled by a worker. The pulley and hook are frictionless and massless.

Draw (describe) the pulley's free-body diagram, explain why the two rope tensions have equal magnitude T, and find T, the worker's pull, from equilibrium of the pulley. Verify with the horizontal equation. [6]

3.2 A lamp of weight 120 N hangs from point O. Cable OA runs horizontally to a wall; strut OB is pinned at O and at the wall, rising at 50° above horizontal, with the lamp hung directly below O.

Explain why the three forces at O must be concurrent, then find the cable tension and the strut force using axes along and perpendicular to the strut. Check both answers with horizontal and vertical equations. [7]