# 1.1 Vectors and forces

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

**KEY WORDS** force • magnitude • direction • resultant • line of action • centre of gravity

## Objective

Resolve forces into components, combine forces into resultants, and express answers in both component and magnitude-direction forms. Every answer is checked by rebuilding the magnitude or by resolving in a second coordinate frame.

## 1 Key knowledge and worked examples

### ■ Resolve, combine, and always rebuild

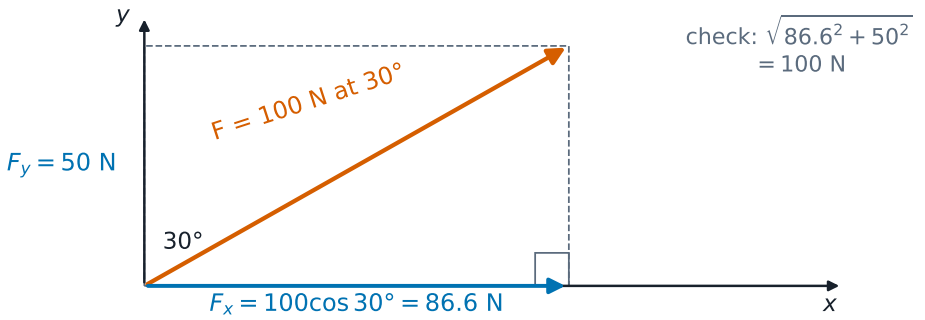
A **force** is a vector with **magnitude** and **direction**. Along axes at angle $\theta$:

$$F_x = F\cos\theta \qquad F_y = F\sin\theta$$

The **resultant** of several forces is their vector sum: add components, then $R = \sqrt{\sum F_x^2 + \sum F_y^2}$, $\theta_R = atan(\sum F_y / \sum F_x)$. The free check is to rebuild the magnitude from the components – it must return the original number.

**Worked example.** A 100 N force at 30°: $F_x = 86.6N$, $F_y = 50N$, and $\sqrt{86.6^2 + 50^2} = 100N$ exactly. Two perpendicular forces of 30 N and 40 N give a resultant of 50 N along the 3-4-5 diagonal – a triangle worth knowing on sight.

**Components are the sides; the force is the diagonal**




### ■ Line of action: slide, never step off

On a **rigid body** a force may be **moved along its line of action** without changing its effect – but never sideways off that line, and never onto a different body. The slide rule is what lets us line up forces at a common point before adding them.

Weights always act at the **centre of gravity**, vertically down, with `W = mg` ($g = 9.81 m/s^2$): a 50 kg block weighs 490.5 N whatever its shape.

### ■ Worked example: the tilted-frame check

Forces of 300 N along +x and 400 N along +y have resultant 500 N at 53.1°. Now resolve the same pair in axes tilted 45° (u at 45°, v at −45°):

- Along u: `(300 + 400) cos 45° = 495.0 N`
- Along v: `(400 - 300) cos 45° = 70.7 N`
- Rebuild: $\sqrt{495.0^2 + 70.7^2} = 500N$ – the same resultant, as it must be.

Components depend on the frame; the force does not. That invariance is the check that catches sign slips better than repeating the first calculation.

## 2 Practice

**2.1** Resolve a 250 N force directed 40° above the horizontal into horizontal and vertical components, and verify by rebuilding the magnitude. [3]

**2.2** A sled is pulled by 60 N east and pushed by 80 N north. Find the resultant's magnitude and direction (from east), and name the triangle you used. [3]

**2.3** Three forces act at a point: 200 N along +x, 150 N along +y, and 100 N at 210° (measured counterclockwise from +x). Find the resultant in component form, then as magnitude and direction. [4]

## 3 Exam-style questions

**3.1** A 400 N load is held by two symmetric cables, each at 25° from the vertical.

Find the tension in each cable. Then explain, with one calculation, what happens to the tension as the cables are made more horizontal (angle increased), and state why the vertical components carry the load while the horizontal ones must cancel. [6]

**3.2** Forces of 300 N along +x and 400 N along +y act on a bracket.

Find the resultant as magnitude and direction. Then resolve the same two forces in axes rotated 45° (u at 45°, v perpendicular at −45°), give both components, and show that the rebuilt magnitude matches your first answer. State what this check demonstrates about components versus forces. [6]