UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

9.3 Static and fatigue criteria

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS fatigue • S-N curve S-N • endurance limit • Marin factors • Goodman line

Objective

Correct a test-bar endurance limit for a real part, read strength from the S-N line, and check a shaft against fatigue with the Goodman line and against first-cycle yield.

1 Key knowledge and worked examples

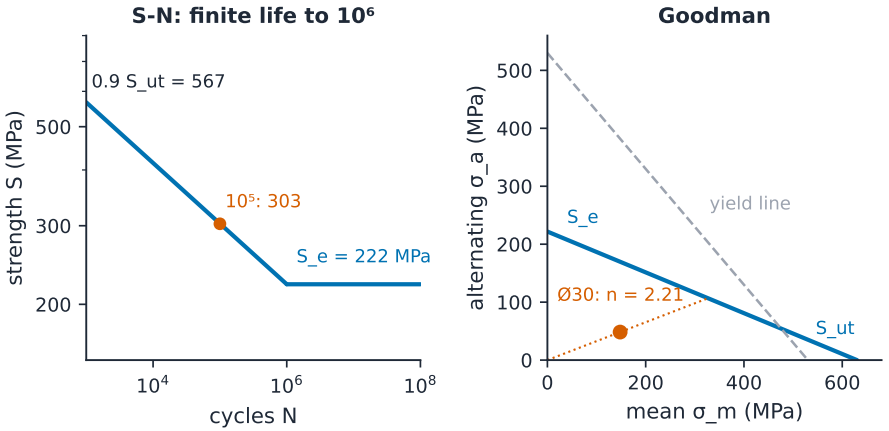
■ **From test bar to real part**

- **Fatigue** : steel's **S-N curve** S-N flattens at the **endurance limit** , $S'_e = 0.5S_{ut}$ (up to 1400 MPa).
- **Marin factors** : $S_e = k_a k_b S'_e$, machined $k_a = 4.51 S_{ut}^{-0.265}$, size $k_b = 1.24 d^{-0.107}$ (2.79 to 51 mm).
- S-N line from $0.9S_{ut}$ at 10^3 to S_e at 10^6 cycles.
- **Goodman line** : $1/n = \sigma_a/S_e + \sigma_m/S_{ut}$; yield: $n_y = S_y/(\sigma_a + \sigma_m)$ by von Mises.

■ **Worked example. The 30 mm shaft**

C45: $S_{ut} = 630$ MPa, $S_y = 530$ MPa, machined, $d = 30$ mm.

- $k_a = 4.51 \times 630^{-0.265} = 0.817$; $k_b = 1.24 \times 30^{-0.107} = 0.862$.
- $S_e = k_a k_b S'_e = 0.817 \times 0.862 \times 315 =$ **222** MPa.
- With $\sigma_a = 48.4$ and $\sigma_m = 148.0$ MPa: $1/n = 48.4/222 + 148.0/630$, so $n =$ **2.21**; yield $n_y = 530/\sqrt{48.4^2 + 148.0^2} = 3.40$. Fatigue governs.



2 Practice

C45 steel, $S_{ut} = 630$ MPa, $S_y = 530$ MPa, machined finish.

2.1 Find k_a, k_b and S_e for a 40 mm shaft (three decimals, three decimals, nearest MPa). [4]

2.2 Using that S_e, find the fully reversed strength at 10^4 cycles from the S-N line (nearest MPa). [3]

3 Exam-style questions

3.1 At its shoulder the 40 mm shaft carries $\sigma_a = 60$ MPa (bending, K_f included) and a von Mises mean $\sigma_m = 120$ MPa. Find the Goodman factor of safety and the first-cycle yield factor of safety (two decimals). Which governs, and what would you change first to raise it? [6]

Solutions

2.1

- $S'_e = 0.5 S_{ut} = 315$ MPa.
- $k_a = 4.51 S_{ut}^{-0.265} = 4.51 \times 630^{-0.265} = \mathbf{0.817}$; $k_b = 1.24 d^{-0.107} = 1.24 \times 40^{-0.107} = \mathbf{0.836}$.
- $S_e = k_a k_b S'_e = 0.817 \times 0.836 \times 315 = \mathbf{215}$ MPa.

2.2

- The line is straight on log axes: each decade from 10^3 multiplies S by $(S_e/0.9S_{ut})^{1/3} = (215/567)^{1/3} = 0.724$.
- At 10^4: $S = 567 \times 0.724 = \mathbf{410}$ MPa.

3.1

- Goodman: $1/n = \sigma_a/S_e + \sigma_m/S_{ut} = 60/215 + 120/630 = 0.469$, so $n = \mathbf{2.13}$.
- Yield: $n_y = S_y/\sqrt{\sigma_a^2 + \sigma_m^2} = 530/\sqrt{60^2 + 120^2} = \mathbf{3.95}$.
- Fatigue governs. Lower K_f first, with a larger shoulder fillet radius, before resorting to a bigger shaft or a stronger steel.