7.6 External flow

Name: _____ Class: _____ Date: _____ **Total: 18 marks**

KEY WORDS boundary layer • skin friction

Objective

Estimate boundary-layer thickness and skin-friction drag on a flat plate, the drag on a blunt body from its drag coefficient, and the power a vehicle spends against the air.

1 Key knowledge and worked examples

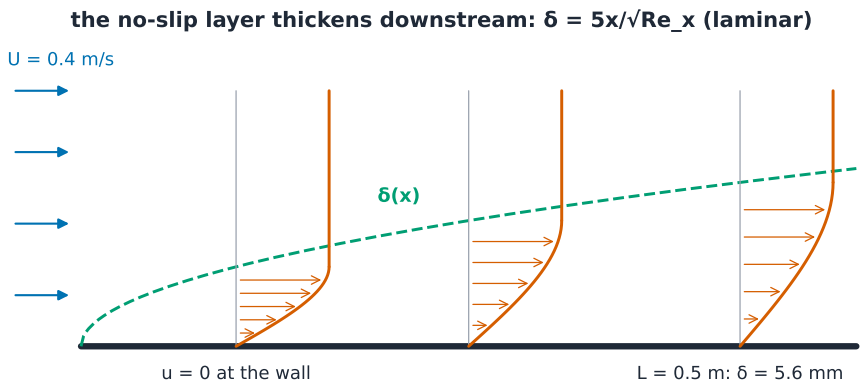
Thin layers, two kinds of drag

- **Boundary layer** on a flat plate (laminar): $\delta = 5x/\sqrt{Re_x}$, turning turbulent near $Re_x = 5 \times 10^5$.
- **Skin friction** (laminar, Blasius): $C_f = 1.328/\sqrt{Re_L}$ and $F = C_f \, \frac{1}{2}\rho U^2 A_{wet}$ per side.
- Blunt bodies: $F_D = C_D \, \frac{1}{2}\rho U^2 A_{front}$; a cylinder has $C_D \approx 1.2$ below the drag crisis.
- Power against drag: $P = F_D U$, so it grows as U^3.

Worked example. The plate in water

A plate 0.5 m long and 0.3 m wide in water at 0.4 m/s.

- $Re_L = UL/\nu = 0.4 \times 0.5/(1.004 \times 10^{-6}) = 1.99 \times 10^5$: laminar throughout.
- Edge thickness: $\delta = 5L/\sqrt{Re_L} = 5 \times 0.5/\sqrt{1.99 \times 10^5} = \mathbf{5.6}$ mm.
- $C_f = 1.328/\sqrt{Re_L} = 0.00298$, so one side carries $F = C_f \frac{1}{2}\rho U^2 A = \mathbf{0.036}$ N.



2 Practice

Air $\rho = 1.204$ kg/m^3, $\nu = 1.516 \times 10^{-5}$ m^2/s.

2.1 Air at 3 m/s flows along a plate 0.8 m long. Find Re_L (three significant figures), say whether the layer stays laminar, and find its thickness at the trailing edge (one decimal, mm). [4]

2.2 The plate of 2.1 is 0.5 m wide and the air flows over both sides. Find the skin-friction drag (three significant figures, N). [4]

2.3 A flagpole 0.08 m in diameter and 6 m tall stands in a 20 m/s wind. Check Re, then find the drag with $C_D = 1.2$ (one decimal, N) and the bending moment at its base (nearest N · m). [5]

3 Exam-style questions

3.1 A car has $C_D = 0.30$ and frontal area 2.2 m^2. Find its drag and the power to overcome it at 30 m/s (nearest newton, one decimal kW). By what factor does that power rise at 35 m/s, and what is it then? [5]

Solutions

2.1

- $Re_L = UL/\nu = 3 \times 0.8/(1.516 \times 10^{-5}) = \mathbf{1.58 \times 10^5}$, below 5×10^5: **laminar**.
- $\delta = 5L/\sqrt{Re_L} = 5 \times 0.8/\sqrt{1.58 \times 10^5} = \mathbf{10.1}$ mm.

2.2

- $C_f = 1.328/\sqrt{Re_L} = 1.328/\sqrt{1.583 \times 10^5} = 0.00334$.
- One side: $F = C_f \frac{1}{2}\rho U^2 A = 0.00334 \times 0.5 \times 1.204 \times 3^2 \times 0.4 = 0.00723$ N.
- Both sides: $F = 2 \times 0.00723 = \mathbf{0.0145}$ N.

2.3

- $Re = UD/\nu = 20 \times 0.08/(1.516 \times 10^{-5}) = 1.06 \times 10^5$, below the drag crisis, so $C_D = 1.2$ holds.
- $F_D = C_D \frac{1}{2}\rho U^2 A = 1.2 \times 0.5 \times 1.204 \times 20^2 \times (0.08 \times 6) = \mathbf{138.7}$ N.
- Uniform wind acts at mid-height: $M = F_D \times 3 = \mathbf{416}$ N $\cdot$ m.

3.1

- $F_D = C_D \frac{1}{2}\rho U^2 A = 0.30 \times 0.5 \times 1.204 \times 30^2 \times 2.2 = \mathbf{358}$ N.
- $P = F_D U = 357.6 \times 30 = \mathbf{10.7}$ kW.
- $P \propto U^3$: factor $(35/30)^3 = \mathbf{1.59}$, so $P = 1.59 \times 10.7 = \mathbf{17.0}$ kW.