7.4 Dimensional analysis

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS dimensional analysis • Reynolds number • dynamic similarity

Objective

Count and form dimensionless groups, compute a Reynolds number, and run a model test with dynamic similarity, scaling the force back to full size.

1 Key knowledge and worked examples

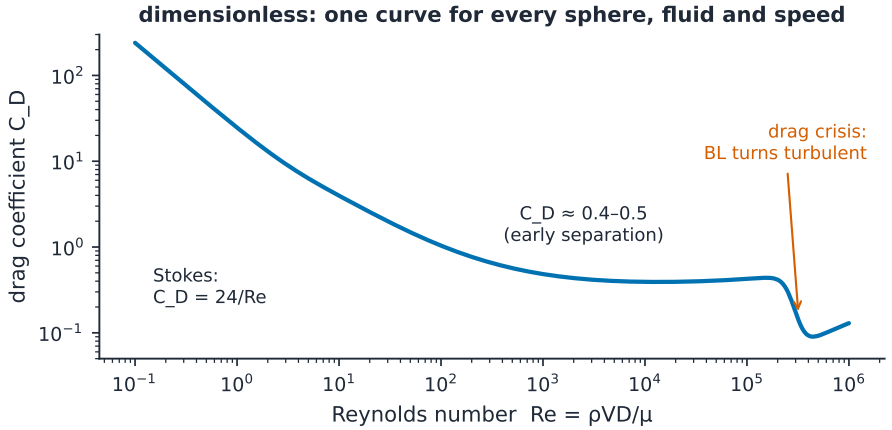
■ **Few groups, one curve**

- **Dimensional analysis** : n variables in k dimensions give $n-k$ independent groups.
- Drag on a body: $C_D = F/(\frac{1}{2}\rho V^2 A) = \phi(Re)$, with the **Reynolds number** $Re = VL/\nu$.
- **Dynamic similarity** : equal Re in model and prototype gives equal C_D, so $F_p = F_m(\rho_p/\rho_m)(V_p/V_m)^2(L_p/L_m)^2$.
- Pipes: laminar below $Re \approx 2300$, turbulent above about 4000.

■ **Worked example. The sensor pod in a water tunnel**

A 0.5 m pod at 20 m/s in air ($\nu = 1.516\times10^{-5}$ m^2/s) is tested at half scale in water ($\nu = 1.004\times10^{-6}$ m^2/s).

- Equal Re: $V_m L_m/\nu_m = V_p L_p/\nu_p$.
- $V_m = V_p(L_p/L_m)(\nu_m/\nu_p) = 20\times2\times(1.004\times10^{-6}/1.516\times10^{-5}) = \mathbf{2.65}$ m/s.



2 Practice

Air $\rho = 1.204$ kg/m^3, $\nu = 1.516\times10^{-5}$ m^2/s; water $\rho = 998$ kg/m^3, $\nu = 1.004\times10^{-6}$ m^2/s.

2.1 The drag F on a smooth sphere depends on ρ, V, D and μ. How many dimensionless groups govern it, and what are they? [3]

2.2 Water flows at 1.2 m/s in a 50 mm pipe. Find Re (nearest hundred) and say whether the flow is laminar or turbulent. [3]

2.3 A car 4.0 m long travels at 25 m/s. A 1:5 model is tested in a water tunnel. Find the tunnel speed needed for dynamic similarity (two decimals, m/s). [3]

3 Exam-style questions

3.1 At the speed of 2.3 the model's drag is 900 N. Find the full-size drag (nearest newton) and the car's drag coefficient if its frontal area is 2.2 m^2 (two decimals). Why would matching Re in a wind tunnel at 1:5 be impractical? [6]

Solutions

2.1

- Five variables (F, ρ, V, D, μ) in three dimensions (M, L, T): $5 - 3 = \mathbf{2}$ groups.
- $F/(\rho V^2 D^2)$, a drag coefficient, and $\rho V D/\mu = Re$.

2.2

- $Re = VD/\nu = 1.2 \times 0.05/(1.004 \times 10^{-6}) = \mathbf{59\,800}$.
- Far above 4000: **turbulent**.

2.3

- Equal Re: $V_m = V_p(L_p/L_m)(\nu_m/\nu_p) = 25 \times 5 \times (1.004 \times 10^{-6}/1.516 \times 10^{-5}) = \mathbf{8.28}$ m/s.

3.1

- Equal C_D: $F_p = F_m(\rho_p/\rho_m)(V_p/V_m)^2(L_p/L_m)^2 = 900 \times (1.204/998) \times (25/8.28)^2 \times 5^2 = \mathbf{248}$ N.
- $C_D = F/(\frac{1}{2}\rho V^2 A) = 248/(0.5 \times 1.204 \times 25^2 \times 2.2) = \mathbf{0.30}$.
- In air the model would need $25 \times 5 = 125$ m/s, a third of the speed of sound, so compressibility would spoil the similarity.

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