7.3 Bernoulli and flow measurement

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS Bernoulli • streamline • discharge coefficient

Objective

Apply Bernoulli along a streamline, and turn a measured pressure difference into a flow or a speed with a venturi, an orifice and a pitot tube.

1 Key knowledge and worked examples

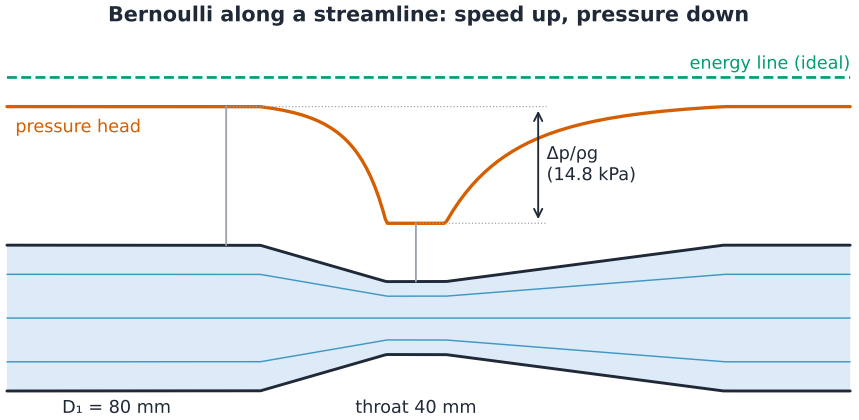
One equation, four conditions

- **Bernoulli** along a **streamline** : $p/\rho g + V^2/2g + z$ is constant, for steady, incompressible, frictionless flow with no pump or turbine between the points.
- Venturi or orifice: $Q = C_d A_2 \sqrt{2\Delta p/(\rho(1-\beta^4))}$, $\beta = D_2/D_1$.
- The **discharge coefficient** : about 0.98 for a venturi, 0.61 for a sharp orifice.
- Pitot tube: $V = \sqrt{2\Delta p/\rho}$.

Worked example. The venturi

80 mm pipe, 40 mm throat ($\beta = 0.5$, $1-\beta^4 = 0.9375$), $\Delta p = 14.8$ kPa.

- Throat area $A_2 = 1.257 \times 10^{-3}$ m^2.
- Ideal: $Q_i = A_2\sqrt{2\Delta p/(\rho(1-\beta^4))} = 1.257 \times 10^{-3}\sqrt{2 \times 14\,803/(998 \times 0.9375)} = 7.07$ L/s.
- Real: $Q = C_d Q_i = 0.98 \times 7.07 = \mathbf{6.93}$ L/s.



2 Practice

Water $\rho = 998$ kg/m^3, air $\rho = 1.204$ kg/m^3, $g = 9.81$ m/s^2.

2.1 An open tank drains through a sharp 20 mm orifice 2.0 m below the surface. Find the ideal jet speed (two decimals, m/s) and the flow with $C_d = 0.61$ (two decimals, L/s). [4]

2.2 A pitot-static tube in an air duct reads 300 Pa. Find the air speed (one decimal, m/s). [3]

2.3 A venturi narrows a 100 mm pipe to a 50 mm throat and reads $\Delta p = 20$ kPa. Find the ideal flow and the real flow with $C_d = 0.98$ (two decimals, L/s). [4]

3 Exam-style questions

3.1 An orifice plate with the same $\beta = 0.5$ and the same 20 kPa reading replaces the venturi of 2.3. Find its flow with $C_d = 0.61$ (two decimals, L/s). Explain in two sentences why the same reading means less flow, and why the orifice is still often chosen. [5]

Solutions

2.1

- Bernoulli from the still surface to the jet, both at atmospheric pressure: $V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.0} = \mathbf{6.26}$ m/s.
- $Q = C_d A V = 0.61 \times \pi (0.010)^2 \times 6.26 = \mathbf{1.20}$ L/s.

2.2

- Stagnation at the tip: $\Delta p = \tfrac{1}{2}\rho V^2$, so $V = \sqrt{2\Delta p/\rho} = \sqrt{2 \times 300/1.204} = \mathbf{22.3}$ m/s.

2.3

- $A_2 = \pi (0.025)^2 = 1.963 \times 10^{-3}$ m^2; $1 - \beta^4 = 0.9375$.
- $Q_i = A_2 \sqrt{2\Delta p/(\rho(1-\beta^4))} = 1.963 \times 10^{-3} \sqrt{2 \times 20\,000/(998 \times 0.9375)} = \mathbf{12.84}$ L/s.
- $Q = 0.98 \times 12.84 = \mathbf{12.58}$ L/s.

3.1

- $Q = C_d Q_i = 0.61 \times 12.84 = \mathbf{7.83}$ L/s.
- The jet contracts after the sharp plate (the vena contracta) and loses energy in eddies, so less flow gives the same pressure drop.
- An orifice is cheap, short and easy to change; the venturi's gentle cone recovers pressure but costs length and money.