7.2 Conservation and control volumes

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS continuity • control volume

Objective

Use continuity and the steady momentum equation on a control volume to find jet speeds, jet forces, nozzle bolt loads and bend anchor forces.

1 Key knowledge and worked examples

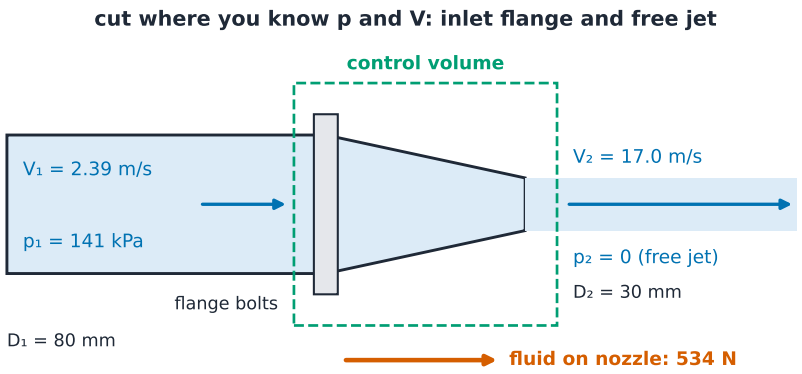
■ **Draw the boundary, count what crosses it**

- **Continuity** for a liquid: $Q = V_1 A_1 = V_2 A_2$.
- Momentum on a **control volume** : $\sum F = \dot{m}(V_2 - V_1)$, component by component, with the pressure forces on the cut faces inside $\sum F$.
- The pipe feels the reverse of the force on the fluid.

■ **Worked example. The nozzle bolts**

$Q = 12$ L/s from an 80 mm pipe into a 30 mm free jet.

- Continuity: $V_1 = Q/A_1 = 0.012/(5.027 \times 10^{-3}) = 2.39$ m/s; $V_2 = Q/A_2 = 0.012/(7.069 \times 10^{-4}) = 17.0$ m/s.
- Bernoulli to the jet ($p_2 = 0$ gauge): $p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) = 141$ kPa.
- Momentum: $p_1 A_1 + F = \rho Q(V_2 - V_1)$, so $F = 174.7 - 708.6 = -533.9$ N: the bolts carry **534** N in tension.



2 Practice

Water $\rho = 998$ kg/m^3; steady flow; losses neglected.

2.1 $Q = 8$ L/s flows in a 60 mm pipe and leaves through a 25 mm nozzle. Find the speed in the pipe and in the jet (one decimal and two decimals, m/s). [3]

2.2 The jet of 2.1 hits a flat plate square on and leaves sideways. Find the force on the plate (nearest newton). [3]

2.3 For the nozzle of 2.1, find the inlet gauge pressure (one decimal, kPa) and the force in the flange bolts (nearest newton), and say whether they are in tension or compression. [5]

3 Exam-style questions

3.1 The same 8 L/s turns through 90° in the 60 mm pipe at 200 kPa gauge. Find the force from each end (pressure plus momentum, one decimal, N), the resultant anchor force (nearest newton) and its direction. Which term dominates, and what does that mean for thrust blocks? [5]

Solutions

2.1

- $A_1 = \pi(0.030)^2 = 2.827 \times 10^{-3}$ m^2; $V_1 = Q/A_1 = 0.008/(2.827 \times 10^{-3}) = \mathbf{2.8}$ m/s.
- $A_2 = \pi(0.0125)^2 = 4.909 \times 10^{-4}$ m^2; $V_2 = Q/A_2 = \mathbf{16.30}$ m/s.

2.2

- All the jet's momentum normal to the plate is destroyed: $F = \rho Q V_2 = 998 \times 0.008 \times 16.30 = \mathbf{130}$ N.

2.3

- Bernoulli, $p_2 = 0$ gauge: $p_1 = \tfrac{1}{2}\rho(V_2^2 - V_1^2) = 0.5 \times 998 \times (16.30^2 - 2.83^2) = \mathbf{128.5}$ kPa.
- Momentum: $p_1 A_1 + F = \rho Q(V_2 - V_1)$, with $p_1 A_1 = 363.4$ N and $\rho Q(V_2 - V_1) = 107.5$ N.
- $F = 107.5 - 363.4 = -255.9$ N on the fluid, so the fluid pushes the nozzle downstream: the bolts carry **256** N in **tension**.

3.1

- Each end: $pA + \rho Q V = 200\,000 \times 2.827 \times 10^{-3} + 998 \times 0.008 \times 2.83 = 565.5 + 22.6 = \mathbf{588.1}$ N.
- The two are at right angles: $R = \sqrt{2} \times 588.1 = \mathbf{832}$ N, at 45° into the outside of the bend.
- Pressure is 96% of each term, so even slow, high-pressure lines need thrust blocks at bends.