7.1 Fluid properties and statics

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS hydrostatics • centre of pressure • viscosity

Objective

Find pressure at depth, the force on a submerged gate and where it acts, a manometer's pressure difference, and the shear in a thin oil film.

1 Key knowledge and worked examples

■ The statics toolkit

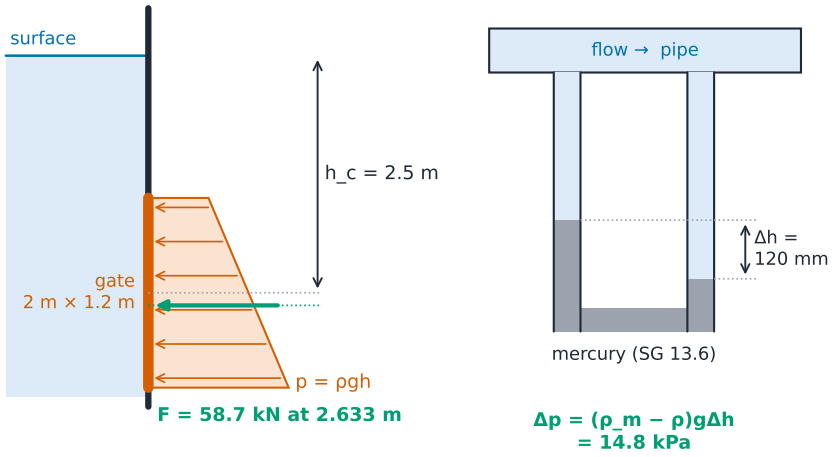
Hydrostatics is exact: a fluid at rest carries no shear.

- Pressure at depth: $p = p_0 + \rho g h$ (gauge when $p_0 = 0$).
- Force on a plane gate: $F = \rho g h_c A$, acting at the **centre of pressure** $y_{cp} = y_c + I_c/(y_c A)$, below the centroid.
- Manometer: $\Delta p = (\rho_m - \rho) g \Delta h$.
- Viscous shear: $\tau = \mu\, du/dy$, with **viscosity** μ.

■ Worked example. The tank gate

A vertical gate 1.2 m wide and 2 m high, top edge 1.5 m down: $h_c = 2.5$ m and $A = 2.4$ m^2.

- Force: $F = \rho g h_c A = 998 \times 9.81 \times 2.5 \times 2.4 = \mathbf{58.7}$ kN.
- $I_c = bh^3/12 = 1.2 \times 2^3/12 = 0.8$ m^4.
- Centre of pressure: $y_{cp} = y_c + I_c/(y_c A) = 2.5 + 0.8/(2.5 \times 2.4) = \mathbf{2.633}$ m, 133 mm below the centroid.



2 Practice

Water $\rho = 998$ kg/m^3, $g = 9.81$ m/s^2, mercury SG 13.6, atmosphere 101.3 kPa.

2.1 A water tank is 4.0 m deep. Find the gauge pressure at the bottom (one decimal, kPa) and the absolute pressure there (one decimal, kPa). [3]

2.2 A vertical rectangular gate 1.0 m wide and 1.5 m high has its top edge 2.0 m below the surface. Find the resultant force (one decimal, kN) and the depth of the centre of pressure (three decimals, m). [5]

2.3 A mercury U-tube across a water pipe fitting reads $\Delta h = 80$ mm. Find the pressure difference (two decimals, kPa). [3]

3 Exam-style questions

3.1 A plate of area 0.4 m^2 slides at 1.5 m/s on an oil film 0.8 mm thick with $\mu = 0.29$ Pa $\cdot$ s. Assume a linear velocity profile. Find (a) the shear stress, (b) the force needed to drag the plate and (c) the power dissipated in the film. [5]

Solutions

2.1

- Gauge pressure from depth: $p = \rho g h = 998 \times 9.81 \times 4.0 = \mathbf{39.2}$ kPa.
- Absolute adds the atmosphere: $p_{abs} = 101.3 + 39.2 = \mathbf{140.5}$ kPa.

2.2

- Centroid depth $h_c = h_{top} + h/2 = 2.0 + 0.75 = 2.75$ m; $A = 1.5$ m^2.
- $F = \rho g h_c A = 998 \times 9.81 \times 2.75 \times 1.5 = \mathbf{40.4}$ kN.
- $I_c = b h^3/12 = 1.0 \times 1.5^3/12 = 0.281$ m^4; $y_{cp} = y_c + I_c/(y_c A) = 2.75 + 0.281/(2.75 \times 1.5) = \mathbf{2.818}$ m.

2.3

- The water columns cancel except over Δh: $\Delta p = (\rho_m - \rho) g \Delta h = (13.6 - 1) \times 998 \times 9.81 \times 0.08 = \mathbf{9.87}$ kPa.

3.1

- (a) Linear profile, so $du/dy = V/h$: $\tau = \mu V/h = 0.29 \times 1.5/0.0008 = \mathbf{544}$ Pa.
- (b) $F = \tau A = 543.75 \times 0.4 = \mathbf{218}$ N.
- (c) $P = FV = 217.5 \times 1.5 = \mathbf{326}$ W, all turned to heat in the oil.