UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

6.6 Buckling and failure criteria

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS buckling • eccentric loading

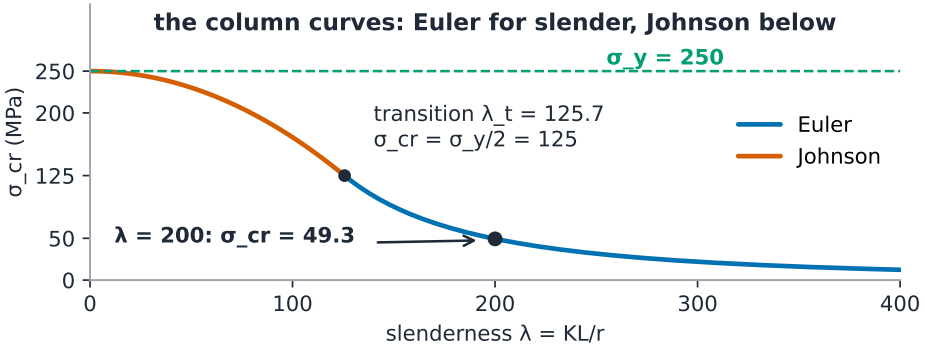
Objective

Decide the buckling regime from the slenderness, apply Euler with a safety factor, and judge a stress state by von Mises.

1 Key knowledge and worked examples

■ The two curves and the verdict

Euler $P_{cr} = \pi^2 EI/(KL)^2$ for $\lambda > \lambda_t = 125.7$; Johnson below; design at P_{cr}/n. Von Mises $\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$ judges the principals.



*Euler for slender columns, Johnson below

■ Worked example. The 20 mm strut

1 m pinned: $\lambda = 200$, $P_{cr} = 15.5$ kN, $\sigma_{cr} = 49.3$ MPa; at $n = 2.5$, $P_{allow} = 6.2$ kN. Von Mises on (109.1, 30.9): $\sigma' = 97.4$ MPa - no yield.

2 Practice

A steel rod of $d = 25$ mm, $L = 1.5$ m, pinned at both ends: $E = 200$ GPa, $\sigma_y = 250$ MPa, $r = 6.25$ mm, $I = 1.917 \times 10^{-8}$ m^4, $A = 4.909 \times 10^{-4}$ m^2.

2.1 The slenderness ratio (integer) and which curve - Euler or Johnson - governs, with the reason. [3]

2.2 The Euler buckling load (one decimal, kN). [4]

2.3 The allowable load at a design factor $n = 2.0$ (one decimal, kN). [3]

2.4 A separate point carries principals $\sigma_1 = 60$ and $\sigma_2 = 20$ MPa: the von Mises stress (one decimal, MPa) and, in one sentence, what it is compared against to decide on yielding. [4]

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Solutions

2.1

- $\lambda = KL/r = 1500/6.25 = \mathbf{240}$.
- $240 > \lambda_t = 125.7$, so Euler governs.

2.2

- $P_{cr} = \pi^2 EI/L^2 = 9.870 \times 200 \times 10^9 \times 1.917 \times 10^{-8}/2.25 = \mathbf{16.8}$ kN.

2.3

- $P_{allow} = P_{cr}/n = 16.8/2.0 = \mathbf{8.4}$ kN.

2.4

- $\sigma' = \sqrt{60^2 - 60 \times 20 + 20^2} = \sqrt{2800} = \mathbf{52.9}$ MPa.
- It is compared with the yield stress in uniaxial tension ($\sigma_y = 250$ MPa): below, no yielding by the distortion-energy criterion.

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