UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

6.4 Beam stresses and deflection

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS flexure formula • deflection

Objective

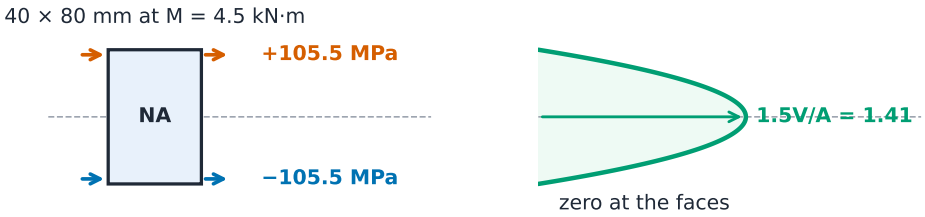
Price the beam: bending stress by the flexure formula and the section modulus, transverse shear, and deflection by the standard cases.

1 Key knowledge and worked examples

■ The stress laws and the cases

$\sigma = Mc/I = M/S$; $\tau_{max} = 1.5V/A$ for rectangles; $\delta = PL^3/48EI$ (mid-span), $PL^3/3EI$ (tip).

bending is linear in depth, shear is parabolic



*Bending linear in depth, shear parabolic

■ Worked example. The 40 x 80 arm

At $M = 4.5$ kN · m: $\sigma = 105.5$ MPa with $S = 42.7 \times 10^{-6}$ m^3; $\tau_{max} = 1.41$ MPa; mid-span $\delta = 9.89$ mm; tip case $\delta = 1.35$ mm.

2 Practice

A steel beam ($E = 200$ GPa) with the 50×100 mm rectangular section: $I = 4.167 \times 10^{-6}$ m^4, $c = 0.05$ m, $A = 0.005$ m^2.

2.1 Simply supported over 4 m with a central $P = 8$ kN: the maximum bending stress (one decimal, MPa) and the section modulus (one decimal, 10^{-6} m^3). [4]

2.2 The same beam at its maximum shear force: the maximum transverse shear stress (one decimal, MPa). [3]

2.3 The same beam and load: the mid-span deflection (one decimal, mm). [4]

2.4 The same section as a 1.5 m cantilever with $P = 1.2$ kN at the tip: the tip deflection (two decimals, mm), and the single cheapest change that would stiffen it. [3]

Solutions

2.1

- $M_{max} = PL/4 = \mathbf{8.0}$ kN · m; $\sigma = Mc/I = 8000 \times 0.05/(4.167 \times 10^{-6}) = \mathbf{96.0}$ MPa.
- $S = I/c = 4.167 \times 10^{-6}/0.05 = \mathbf{83.3} \times \mathbf{10^{-6}}$ m^3.

2.2

- $V_{max} = P/2 = 4$ kN; $\tau_{max} = 1.5V/A = 1.5 \times 4000/0.005 = \mathbf{1.2}$ MPa.

2.3

- $EI = 200 \times 10^9 \times 4.167 \times 10^{-6} = 8.333 \times 10^5$ N m^2.
- $\delta = PL^3/(48EI) = 512\,000/(4.000 \times 10^7) = \mathbf{12.8}$ mm.

2.4

- $\delta = PL^3/(3EI) = 4050/(2.500 \times 10^6) = \mathbf{1.62}$ mm.
- Increase the section depth: I carries h^3 - double the depth, one eighth the sag.

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