UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

5.2 First law for closed systems

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS closed system • boundary work • adiabatic process

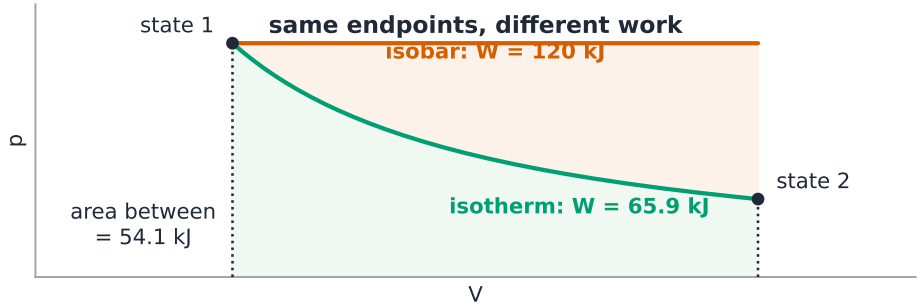
Objective

Keep the closed-system ledger: boundary work as area under the path, the first law with one sign convention, and the adiabatic relation.

1 Key knowledge and worked examples

■ The ledger

$\Delta U = Q - W$ (work by the system positive); $W = \int_1^2 p\,dV$ is the area under the path; reversible adiabatic: $pV^\gamma =$ constant with $\gamma = 1.4$ for air.



*The area under each path is that path's work

■ Worked example. Three paths, one pair of states

Isobaric to triple volume: $W = 200 \times 0.6 = 120$ kJ; isothermal: $W = 200 \times 0.3 \times \ln 3 = 65.9$ kJ ✓ - 54.1 kJ apart, exactly the area between the curves.

2 Practice

Air (closed system) starts at $(300\text{ kPa}, 0.2\text{ m}^3)$ with $\gamma = 1.4$.

2.1 Air at 300 kPa, 0.2 m^3 expands isobarically to 0.6 m^3: the boundary work, and the heat transfer if $\Delta U = +84$ kJ (one decimal, kJ). [4]

2.2 The same gas, from 300 kPa, 0.2 m^3 to 0.6 m^3, connected isothermally instead: the boundary work (one decimal, kJ). [3]

2.3 The gas ($\gamma = 1.4$), starting again at 300 kPa, 0.2 m^3, is compressed adiabatically to 0.1 m^3: the final pressure (one decimal, kPa), the work (one decimal, kJ) and the change in internal energy (one decimal, kJ). [5]

2.4 One sentence each: why is boundary work path-dependent, and why does $\Delta U = mc_v\Delta T$ still hold on a non-constant-volume path for an ideal gas? [2]

Solutions

2.1

- $W = p\Delta V = 300 \times (0.6 - 0.2) = \mathbf{120.0}$ kJ.
- $Q = \Delta U + W = 84 + 120.0 = \mathbf{204.0}$ kJ.

2.2

- $W = p_1 V_1 \ln(V_2/V_1) = 300 \times 0.2 \times \ln 3 = \mathbf{65.9}$ kJ.

2.3

- $p_2 = 300 \times (0.2/0.1)^{1.4} = 300 \times 2.639 = \mathbf{791.7}$ kPa.
- $W = (p_2 V_2 - p_1 V_1)/(1 - \gamma) = (79.17 - 60)/(1 - 1.4) = -\mathbf{47.9}$ kJ.
- $Q = 0$, so $\Delta U = -W = +\mathbf{47.9}$ kJ ✓.

2.4

- The integral $W = \int p\,dV$ runs along the chosen curve $p(V)$, so different connective paths enclose different areas between the same endpoints.
- u is a state function of T alone for an ideal gas, so its differential $mc_v\,dT$ is a property relation, valid on every path.

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