3.6 Rigid-body kinetics

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS mass moment of inertia • torque • tipping

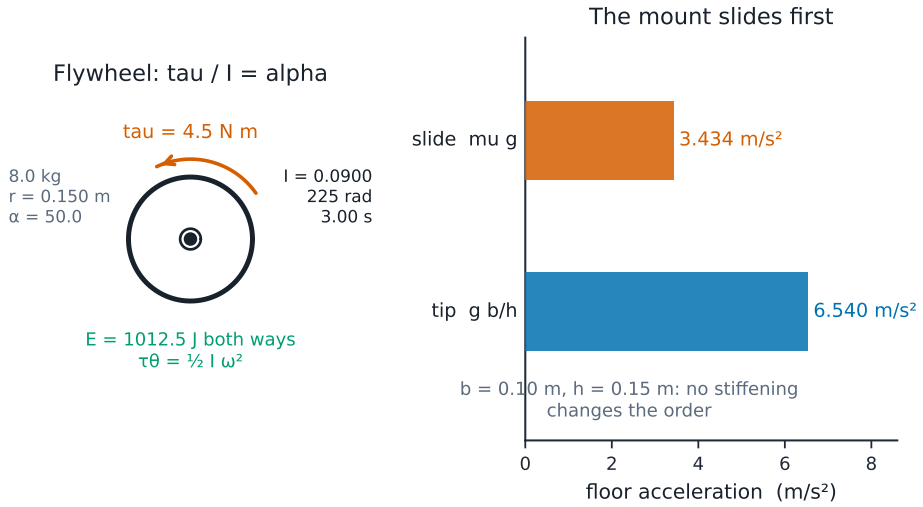
Objective

Compute moments of inertia about a stated axis, drive rotation by torque and energy, and decide between sliding and tipping.

1 Key knowledge and worked examples

■ Rotation tools

Disk about its axle: $I = \frac{1}{2}mr^2$. $\sum \tau = I\alpha$; energy $\frac{1}{2}I\omega^2$; rolling $a = g\sin\beta/(1 + I/mr^2)$; tipping at $a = gb/h$ with half-width b and CG height h.



*Slides first - no stiffening changes the order

■ Worked example. The flywheel file

8.0 kg disk at 0.150 m: $I = 0.0900$, $\alpha = \tau/I = 50.0$ rad/s^2, 225 rad to $\omega = 150$ rad/s storing 1012.5 J $= \tau\theta$ ✓; the mount slides at 3.434 before it tips at 6.540 m/s^2 ✓.

2 Practice

A 6.0 kg flywheel disk of radius 0.20 m is spun by a 3.6 N · m motor torque for 5.0 s from rest.

2.1 The moment of inertia about the axle and the angular acceleration. [3]

2.2 The angular velocity and the angle turned after 5.0 s, and the energy stored (check it against the motor's work). [5]

2.3 The power is cut and the flywheel coasts to rest in 50 s. The friction torque (three decimals), assuming it constant. [3]

2.4 A carton with half-width 0.12 m and centre of mass 0.20 m up stands on a floor with $\mu_s = 0.32$. Which happens first as the floor accelerates - sliding or tipping - and at what value? [4]

exercises.lol exercises.com.cn

Solutions

2.1

- $I = \tfrac{1}{2} m r^2 = \tfrac{1}{2} \times 6.0 \times 0.040 = 0.120$ kg · m^2.
- $\alpha = \tau / I = 3.6 / 0.120 = 30.0$ rad/s^2.

2.2

- $\omega = \alpha t = 150$ rad/s; $\theta = \tfrac{1}{2}\alpha t^2 = 375$ rad.
- $E = \tfrac{1}{2} I \omega^2 = 1350$ J $= \tau\theta = 3.6 \times 375$ ✓.

2.3

- $\alpha_f = -150/50 = -3.00$ rad/s^2, so $\tau_f = 0.120 \times 3.00 = 0.360$ N · m.

2.4

- Slide at $\mu_s g = 3.139$ m/s^2; tip at $gb/h = 9.81 \times 0.12/0.20 = 5.886$ m/s^2.
- $3.139 < 5.886$: it slides first, at 3.14 m/s^2.

exercises.lol exercises.com.cn