3.5 Rigid-body kinematics

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS slider-crank mechanism • instant centre • rolling without slipping

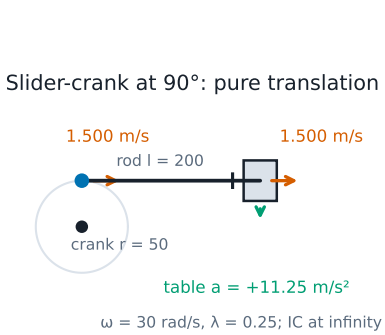
Objective

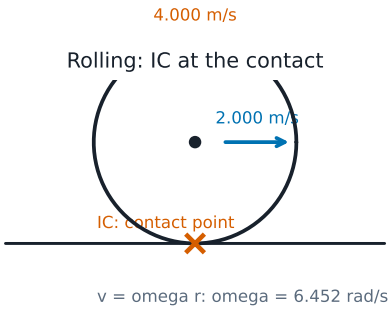
Use constraint geometry first: slider-crank positions, velocities and accelerations, and rolling wheels by instant centre.

1 Key knowledge and worked examples

■ Mechanism kinematics

$x = r\cos\theta + \sqrt{l^2 - r^2\sin^2\theta}$; at $\theta = 90°$: $\dot{x} = -r\omega$, $\ddot{x} = r\omega^2\lambda$ with $\lambda = r/l$. Rolling: $v = \omega r$, top at $2v$, contact at rest.





*The geometry does the differentiating

■ Worked example. The shaker drive

$r = 50$ mm, $l = 200$ mm, $\omega = 30$ rad/s: at quarter-turn $v = 1.500$ m/s and $a = +11.25$ m/s^2; at dead centre -56.25 m/s^2 - a 5:1 range ✓. The 0.310 m wheel at 2.00 m/s: $\omega = 6.452$ rad/s, top 4.000 m/s ✓.

2 Practice

A shaker drive has $r = 40$ mm, $l = 160$ mm, $\omega = 25$ rad/s.

2.1 The piston position at quarter-turn (four decimals), and its velocity magnitude there (three decimals). [3]

2.2 The table acceleration at quarter-turn and at dead centre (two decimals), and the ratio between the extremes. [4]

2.3 A 0.250 m wheel rolls without slipping at 1.80 m/s: the angular velocity (rad/s, one decimal), and the speeds of the contact point, the centre and the top point. [3]

2.4 State where the connecting rod's instant centre is at quarter-turn, and what that implies for the rod's motion. [3]

Solutions

2.1

- $x = \sqrt{l^2 - r^2} = \sqrt{0.160^2 - 0.040^2} = 0.1549$ m.
- $|\dot{x}| = r\omega = 1.000$ m/s.

2.2

- Quarter-turn: $\ddot{x} = r\omega^2\lambda = 0.040 \times 625 \times 0.25 = 6.25$ m/s^2.
- Dead centre: $-r\omega^2(1+\lambda) = -31.25$ m/s^2.
- Ratio 5:1 - the fatigue file's range.

2.3

- $\omega = v/r = 7.2$ rad/s.
- Contact 0, centre 1.80, top 3.60 m/s.

2.4

- At quarter-turn both crank pin and piston move horizontally, so the perpendiculars to the rod's ends are parallel: the instant centre is at infinity.
- The rod is in pure translation: every point at 1.000 m/s.

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