2.5 Vector calculus

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS gradient • divergence theorem

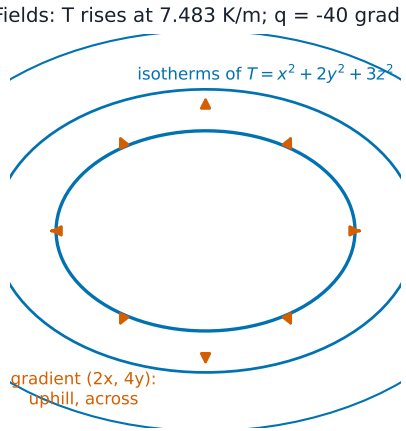
Objective

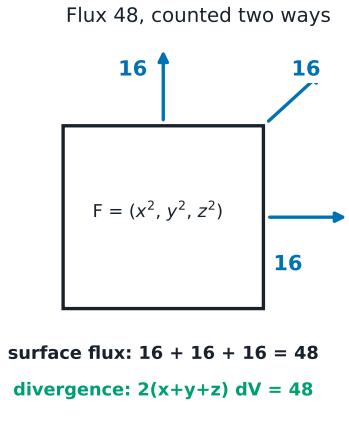
Compute grad/div/curl at a point, and use the theorems to count flux and circulation the cheap way.

1 Key knowledge and worked examples

■ Operators and theorems

∇T uphill; $\nabla \cdot \mathbf{F}$ sources; $\nabla \times \mathbf{G}$ spin. Divergence theorem: closed-surface flux = volume integral; Stokes: loop circulation = curl flux.





Closed surfaces only

■ Worked example. The amplifier

$\nabla T(1,1,1) = (2,4,6)$, $|\nabla T| = 7.483$; $\mathbf{q} = -40\nabla T = (-80,-160,-240)$ W/m2; cube flux $48 = 48$ ✓.

2 Practice

A heater's field is $T = 3x^2 + y^2 + 2z^2$ (k = 40 W/mK), and a flow is $\mathbf{G} = (0, 0, xy)$.

2.1 ∇T at (1, 1, 1), its magnitude, and the heat flux vector with its magnitude (one decimal). [4]

2.2 The flux of $\mathbf{F} = (x^2, y^2, z^2)$ through the cube $[1,3]^3$: by faces, and by the divergence theorem. [4]

2.3 $\nabla \times \mathbf{G}$ at (1, 1, 2). [3]

2.4 The grille has one panel missing. State the theorem's status and the standard repair in one line each. [3]

Solutions

2.1

- $\nabla T = (6x, 2y, 4z) = (6, 2, 4)$; $|\nabla T| = 7.483$ K/m.
- $\mathbf{q} = -40\nabla T = (-240, -80, -160)$ W/m2; $|\mathbf{q}| = 40 \times 7.483 = 299.3$ W/m2.

2.2

- Far faces at 3: $9 \times 4 = 36$ each, 108; near faces at 1: $1 \times 4 = 4$ each, 12: net 96.
- $\int 2(x + y + z)\, dV = 2 \times 6 \times 8 = 96$.

2.3

- $\nabla \times \mathbf{G} = (x, -y, 0)$; at (1,1,2): $(1, -1, 0)$.

2.4

- Open surface: the theorem does not apply directly.
- Repair: patch the hole, apply the theorem, subtract the patch.

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