2.3 Differential equations

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS damped frequency • resonance

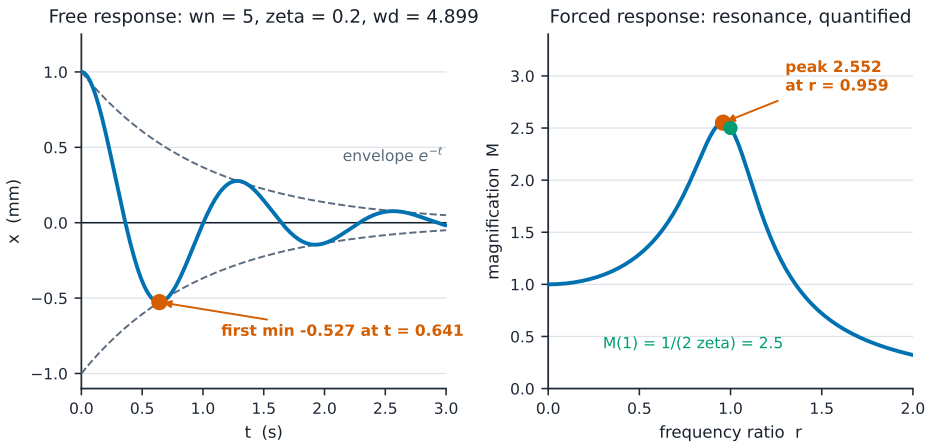
Objective

Characterise a second-order system, verify a free response by substitution, and price the forced response.

1 Key knowledge and worked examples

■ Personality and response

Roots of $ms^2 + cs + k = 0$; $\omega_n = \sqrt{k/m}$, $\zeta = c/(2\sqrt{mk})$, $\omega_d = \omega_n\sqrt{1-\zeta^2}$; magnification $M(r)$ with $M(1) = 1/2\zeta$, peak at $\sqrt{1-2\zeta^2}$.



The envelope is damping made visible

■ Worked example. The certification rig

$m = 1$, $c = 2$, $k = 25$: $\omega_n = 5$, $\zeta = 0.2$, $\omega_d = 4.899$; first minimum -0.527; peak 2.552 at $r = 0.959$ ✓.

2 Practice

A bench rig has $m = 4$ kg, $c = 4$ N s/m, $k = 25$ N/m.

2.1 ω_n, ζ, the response family, and ω_d (four decimals). [4]

2.2 From rest at $x_0 = 1$ mm: the envelope's value at $t = 2$ s, and the time to decay to 1% of the start. [3]

2.3 The floor drives the rig at $r = 1$. The magnification, and the drive amplitude if the till may see at most 5 mm. [3]

2.4 State the two checks that turn a claimed free response into an audited one. [4]

Solutions

2.1

- $\omega_n = \sqrt{25/4} = 2.5$ rad/s; $\zeta = 4/(2\sqrt{100}) = 0.2$ - underdamped.
- $\omega_d = 2.5\sqrt{0.96} = 2.4495$ rad/s.

2.2

- Envelope $e^{-\zeta\omega_n t} = e^{-0.5t}$: at 2 s, $e^{-1} = 0.368$ mm-scale.
- 1% at $0.5t = 4.6$: $t = 9.2$ s.

2.3

- $M(1) = 1/(2 \times 0.2) = 2.5$.
- Drive $\le 5/2.5 = 2$ mm.

2.4

- Substitute x, $\dot{x}$, $\ddot{x}$ from the claimed formula into the ODE: residual at machine precision.
- Check the initial conditions match, and the first-extremum value against $-e^{-\zeta\pi/\sqrt{1-\zeta^2}}$.

exercises.lol exercises.com.cn