2.2 Linear algebra

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS condition number • eigenvalue

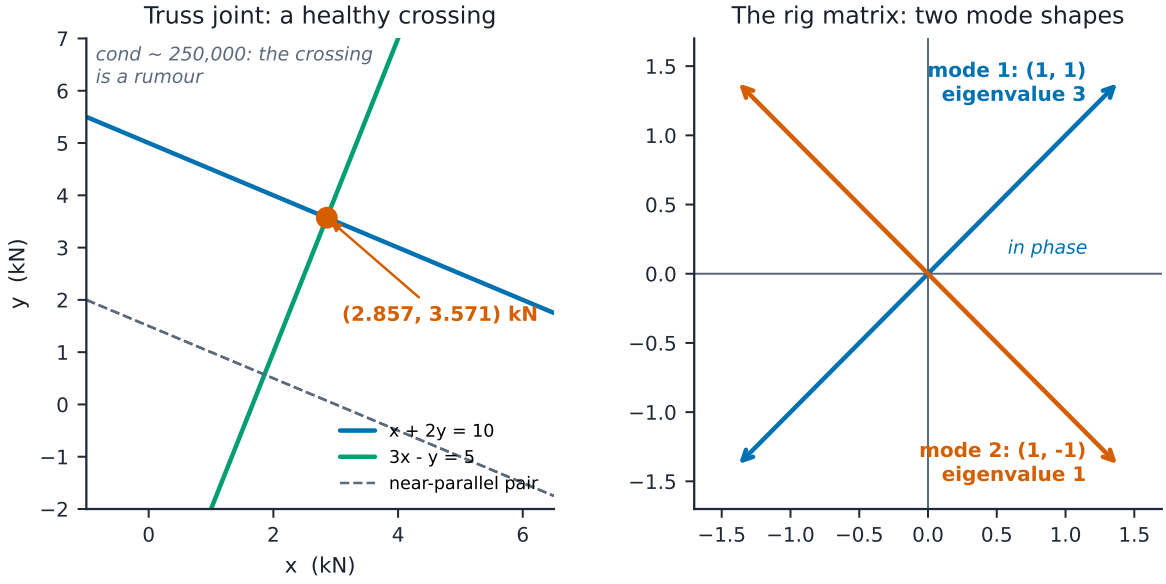
Objective

Solve small systems, interpret conditioning, and use eigenvalues for mode shapes.

1 Key knowledge and worked examples

■ Intersections, modes, conditioning

One intersection per system; eigen-directions of a symmetric matrix are orthogonal; the condition number prices how input noise reaches the answer.



Shallow crossings are rumours

■ Worked example. The rig

Joint (2.857, 3.571) kN; eigenvalues 3 and 1 with modes (1,1), (1,-1); condition numbers 3 against ~250,000 ✓.

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2 Practice

2.1 Solve $2x + 3y = 12$, $x - y = 1$ and verify both equations. [3]

2.2 The matrix with rows (4, 1) and (1, 4): its eigenvalues and eigenvectors. [4]

2.3 The matrix with rows (1, 1) and (1, 1.0001): the condition number (nearest thousand), and what it says about the solved answer. [3]

2.4 An initial displacement (2, 0) of the two-mass rig of 2.2. Express it in the mode basis and state each mode's share. [4]

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Solutions

2.1

- Subtract/eliminate: $x = 3$, $y = 2$.
- Check: $6 + 6 = 12$ and $3 - 2 = 1$.

2.2

- $(4 - \lambda)^2 - 1 = 0$: $\lambda = 5$ and 3.
- $(1, 1)$ for 5; $(1, -1)$ for 3.

2.3

- $\kappa \approx 40{,}000$ (nearest thousand).
- The rows are nearly parallel: a data nudge of 10^{-4} can move the answer by whole units - report the conditioning, not just the solution.

2.4

- $(2, 0) = 1 \cdot (1, 1) + 1 \cdot (1, -1)$ - equal shares.
- Each mode then oscillates at its own frequency; the split is the prediction.