2.1 Calculus for engineering

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS displacement • definite integral

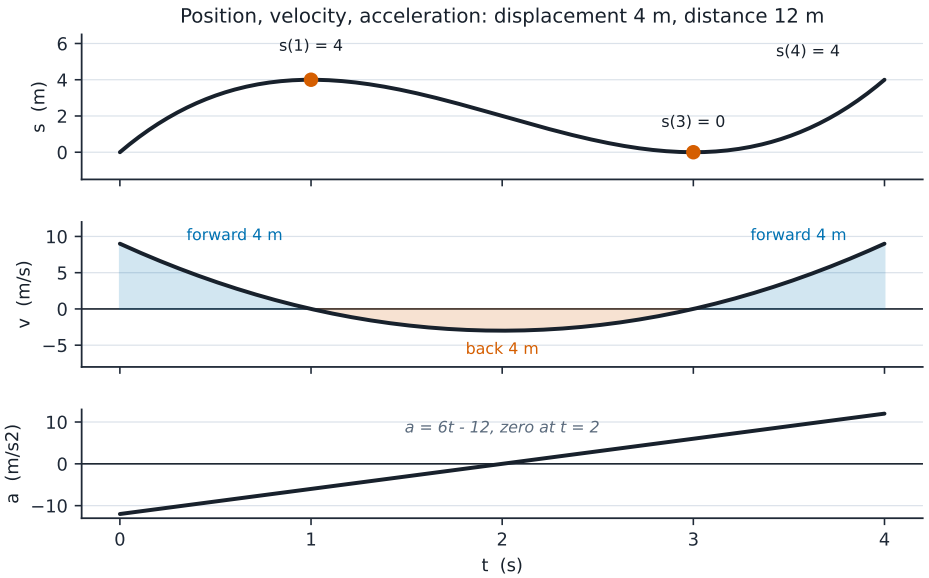
Objective

Extract velocity, acceleration, displacement and distance from a motion trace, and set up and check an area or volume integral.

1 Key knowledge and worked examples

■ Trace and area

$v = ds/dt$, $a = dv/dt$; displacement $= s(b) - s(a)$, distance $= \int |v|\,dt$. Volume of revolution about the x-axis: $\pi \int y^2\,dx$. Dimensional check every line.



The zeroes of v partition the story

■ Worked example. The certification trolley

$s = t^3 - 6t^2 + 9t$: zeroes of v at 1 and 3; displacement 4 m, distance 12 m; the fillet $\pi \times 17.067 = 53.62$ m3, checked by quadrature ✓.

2 Practice

A shuttle on the rig runs $u(t) = 2t^2 - 8t + 6$ m/s for 0 to 4 s.

2.1 The times $u = 0$, the sign of u on each interval, and $u(0)$. [3]

2.2 With $U(0) = 0$: $U(1)$, $U(3)$ and $U(4)$ (three decimals), then the displacement and the distance over 0 to 4 s. [4]

2.3 The acceleration $a(t)$ and its zero. [2]

2.4 The guide fillet $y = x(3 - x)$ is revolved about the x-axis from 0 to 3 m. The volume (two decimals), with the integral set out and one check stated. [5]

Solutions

2.1

- $2t^2 - 8t + 6 = 0$ gives $t = 1$ and $t = 3$; $u(0) = 6$ m/s.
- Positive on (0,1) and (3,4); negative on (1,3).

2.2

- $U(t) = \frac{2}{3}t^3 - 4t^2 + 6t$: $U(1) = 2.667$, $U(3) = 0$, $U(4) = 2.667$ m.
- Displacement $= 2.667$ m; distance $= 2.667 + 2.667 + 2.667 = 8.000$ m.

2.3

- $a = 4t - 8$, zero at $t = 2$ s.

2.4

- $\pi \int_0^3 (9x^2 - 6x^3 + x^4)dx = \pi(81 - 121.5 + 48.6) = \pi \times 8.1 = 25.45$ m3.
- Check: numeric quadrature agrees to two decimals (or shell agreement about the stated axis).

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