UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

12.5 Digital implementation

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS zero-order hold • Nyquist frequency • aliasing • ADC

Objective

Count the phase a zero-order hold and a computation delay cost, choose a sample rate, spot aliasing, find encoder resolution, and run a Tustin PI update by hand.

1 Key knowledge and worked examples

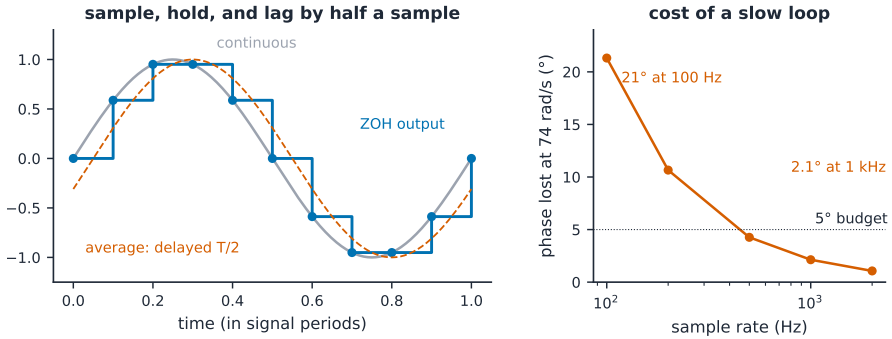
What sampling costs

- An **ADC** samples every T; the **zero-order hold** acts like a delay of $T/2$: phase lag $\omega T/2$ (rad).
- A computation delay τ_d costs $\omega\tau_d$ more.
- Above the **Nyquist frequency** $f_s/2$, a signal at f appears at $|f - kf_s|$: **aliasing**. Filter before the ADC.
- Tustin integral: $I_k = I_{k-1} + \frac{T}{2}(e_k + e_{k-1})$; PI output $u_k = K_p e_k + K_i I_k$. Sample 20 to 30 times the bandwidth.

Worked example. Choosing the sample rate

Valve loop, $\omega_n = 74.4$ rad/s, crossover 48.2 rad/s, $PM = 65.2°$.

- Hold lag at ω_n: $74.4T/2 =$ **2.1**° at 1 kHz, **21**° at 100 Hz.
- Phase margin at crossover: 63.8° at 1 kHz, 51.3° at 100 Hz.
- Encoder: 2000 counts per motor turn × 50 = 100 000 per valve turn, **0.0036**° per count.



2 Practice

The valve loop of sheet 12.3: $\omega_c = 29.8$ rad/s, $PM = 65.2°$.

2.1 Find the hold's phase lag at crossover (two decimals, degrees) and the remaining phase margin when sampling at 500 Hz and at 100 Hz. [4]

2.2 Sampling at 1 kHz, where does noise at 1.2 kHz appear? And noise at 700 Hz? [2]

2.3 A 1000-line encoder (quadrature ×4) on the motor drives the valve through 30 : 1. Find the counts per valve turn and the angle per count (degrees). [2]

3 Exam-style questions

3.1 (a) Find the motor-speed resolution from counting that encoder at 1 kHz (three decimals, rad/s) and say what it implies for derivative action. (b) A 0.5 ms computation delay is added: find its lag at crossover (two decimals, degrees). (c) With $T = 2$ ms, $K_p = 79.4$, $K_i = 365.2$, $I_0 = 0$ and errors $e_0 = 0.10$, $e_1 = 0.08$, $e_2 = 0.06$ rad, find I_1, I_2 and u_2 (three decimals, V). [8]

Solutions

2.1

- Lag $= \omega_c T/2$: at 500 Hz, 29.8×0.001 rad = **1.71**°; at 100 Hz, 29.8×0.005 rad = **8.54**°.
- Margins: $PM' = PM - \omega_c T/2 = 65.2 - 1.7 =$ **63.5**° and $65.2 - 8.5 =$ **56.7**°.

2.2

- $|1200 - 1000| =$ **200** Hz; $|700 - 1000| =$ **300** Hz: both fold below the 500 Hz Nyquist frequency, indistinguishable from real signals.

2.3

- $1000 \times 4 \times 30 =$ **120 000** counts per valve turn.
- $360/120\,000 =$ **0.003**° per count.

3.1

- (a) One count per sample: $2\pi/4000/0.001 =$ **1.571** rad/s at the motor.
- The speed estimate jumps in steps of 1.57 rad/s, so $K_d\dot{e}$ would inject that noise: leave D out or filter heavily.
- (b) $\omega_c \tau_d = 29.8 \times 0.0005 = 0.0149$ rad = **0.85**°.
- (c) $I_1 = I_0 + \frac{T}{2}(e_1 + e_0) = 0.001 \times 0.18 =$ **0.00018**.
- $I_2 = I_1 + \frac{T}{2}(e_2 + e_1) = 0.00018 + 0.001 \times 0.14 =$ **0.00032**.
- $u_2 = K_p e_2 + K_i I_2 = 79.4 \times 0.06 + 365.2 \times 0.00032 =$ **4.881** V.