UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

12.4 Feedback and PID control

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS error • PID controller PID • rate limit • integrator windup • anti-windup

Objective

Find the steady error a load leaves under proportional control, add integral action to remove it, and see how actuator saturation causes integrator windup.

1 Key knowledge and worked examples

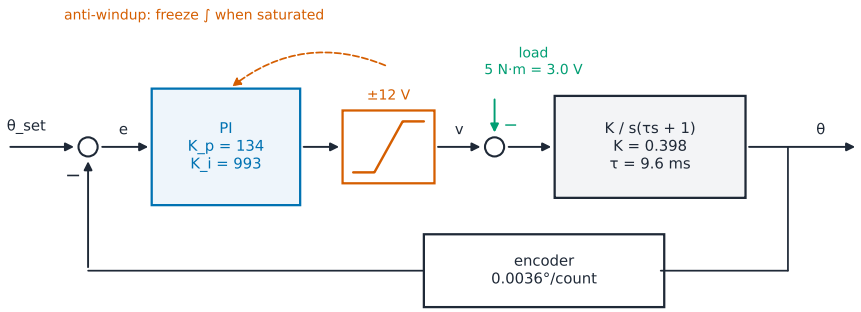
P, I and D, and the limits

- **PID controller** PID : $u = K_p e + K_i \int e\,dt + K_d \dot{e}$, acting on the **error** e.
- A constant disturbance d (volts at the motor) leaves $e = d/K_p$ under P; the integral supplies d with $e = 0$.
- Common start: $K_i = K_p \omega_n / 10$ (zero a decade below ω_n), costing about 9° of phase margin.
- Saturated, the drive moves at its **rate limit** ; the integrator keeps growing: **integrator windup** . **Anti-windup** clamps it.

Worked example. Tuning against a spec

Valve loop, $K_p = 134$ V/rad, $\omega_n = 74.4$ rad/s; a 5 N · m load is 3.0 V at the motor; spec: error $\leq 0.5°$.

- P alone: $e = d/K_p = 3.0/134 = 0.0225$ rad = **1.29°**: fails.
- $K_i = K_p \omega_n / 10 =$ **993** V/(rad · s): error 0.
- A 90° step asks $134 \times 1.571 = 210$ V of 12 V: without anti-windup the valve overshoots to 159°.



2 Practice

The valve drive of sheet 12.3: $K_p = 79.4$ V/rad, $\omega_n = 46.0$ rad/s, plant gain 0.414 rad/s per volt at the valve, 24 V supply. The 8 N · m load needs 7.84 V at the motor.

2.1 Find the steady error under P control (four decimals, rad, and two decimals, degrees). Does it meet a 0.5° spec? [3]

2.2 Choose K_i by the decade rule (one decimal) and give the steady error with it. Why is derivative action usually left out with an encoder? [3]

3 Exam-style questions

3.1 A 90° step is commanded. (a) Find the voltage P control asks for at once (one decimal, V) and say what happens. (b) Find the valve speed at the rate limit under load (two decimals, rad/s) and the time the slew takes. (c) Explain integrator windup during the slew and its effect on arrival. (d) Describe clamping anti-windup and one other remedy. [8]

Solutions

2.1

- $e = d/K_p = 7.84/79.4 = \mathbf{0.0988}$ rad.
- $= \mathbf{5.66}°$: eleven times the 0.5° spec, fails.

2.2

- $K_i = K_p\omega_n/10 = 79.4 \times 46.0/10 = \mathbf{365.2}$ V/(rad · s); steady error 0, since the integral grows until the motor sees 7.84 V with $e = 0$.
- Speed from counting encoder steps is coarse, and $K_d\dot{e}$ amplifies that noise into the drive.

3.1

- (a) $u = K_p e = 79.4 \times 1.571 = \mathbf{124.7}$ V against a 24 V supply: the drive saturates and the loop is open.
- (b) $\omega_{max} = (V_s - d) \times 0.414 = (24 - 7.84) \times 0.414 = \mathbf{6.69}$ rad/s; $1.571/6.69 \approx \mathbf{0.23}$ s plus start-up.
- (c) The error stays large while saturated, so $\int e\,dt$ grows with no effect on u. On arrival the stored integral still pushes, driving the valve well past 90° until it unwinds.
- (d) Clamping: stop integrating while the output is saturated and the error would push it further.
- Or back-calculation (bleed the integral by the saturation excess), or rate-limiting the set point so the drive never saturates.