UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

12.2 Electromechanical systems

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS dc motor • back emf • mechanical time constant

Objective

Find a dc motor's speed per volt, its time constants and stall values, then carry them through a gearbox to the load: top speed, holding voltage and the torque a current limit allows.

1 Key knowledge and worked examples

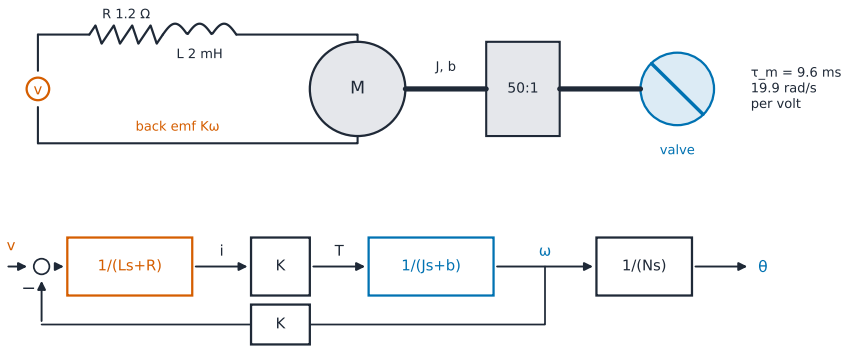
■ **Two constants join the circuit and the rotor**

- **DC motor** : torque $T = Ki$, **back emf** $e = K\omega$ (in SI, $K_t = K_e = K$).
- $\dfrac{\Omega}{V} = \dfrac{K}{(Js+b)(Ls+R)+K^2}$; with small L, gain $K/(bR+K^2)$ and **mechanical time constant** $\tau_m = JR/(bR+K^2)$; electrical $\tau_e = L/R$.
- Stall: $i = V/R$, $T = KV/R$.
- Through a gearbox N, efficiency η: speed $/N$; load torque at the motor $T_L/(N\eta)$.

■ **Worked example. The valve motor**

$R = 1.2\ \Omega$, $L = 2$ mH, $K = 0.05$ N · m/A, $J = 2 \times 10^{-5}$ kg · m^2, $b = 10^{-5}$ N · m · s/rad.

- Gain $= K/(bR+K^2) = 0.05/(1.2\times10^{-5}+0.0025) =$ **19.9** rad/s per volt; 12 V gives 239 rad/s.
- $\tau_m = JR/(bR+K^2) =$ **9.6** ms; $\tau_e = L/R = 1.7$ ms.
- Through $N = 50$: a 90° move takes at least **0.33** s.



2 Practice

A motor has $R = 2\ \Omega$, $L = 3$ mH, $K = 0.08$ N · m/A, $J = 5\times10^{-5}$ kg · m^2, $b = 2\times10^{-5}$ N · m · s/rad, on a 24 V supply.

2.1 Find the steady speed per volt (two decimals, rad/s per V) and the no-load speed at 24 V (nearest rad/s). [3]

2.2 Find τ_m (one decimal, ms) and τ_e, and the stall current and torque. [4]

3 Exam-style questions

3.1 The motor drives a valve through a $30:1$ gearbox with efficiency 0.85. The valve stem needs $8\ \mathrm{N\cdot m}$. (a) Find the motor torque, current and the voltage that just holds the valve (two decimals). (b) Find the valve's top speed with no load and under load at 24 V (two decimals, rad/s), and the shortest 90° move under load (three decimals, s). (c) A 6 A current limit is set: find the valve torque it allows and the mechanical power at the motor shaft while moving under load. [9]

Solutions

2.1

- Gain $= K/(bR + K^2) = 0.08/(4 \times 10^{-5} + 0.0064) =$ **12.42** rad/s per V.
- $\omega = V(\Omega/V) = 24 \times 12.42 =$ **298** rad/s (2850 rpm).

2.2

- $\tau_m = JR/(bR + K^2) = 10^{-4}/0.00644 =$ **15.5** ms.
- $\tau_e = L/R = 3 \times 10^{-3}/2 =$ **1.5** ms.
- Stall: $i = V/R = 12$ A, $T = Ki =$ **0.96** N $\cdot$ m.

3.1

- (a) $T_m = T_L/(N\eta) = 8/(30 \times 0.85) =$ **0.314** N $\cdot$ m.
- $i = T_m/K =$ **3.92** A; at rest no back emf, so $V = iR =$ **7.84** V.
- (b) No load: $298/30 =$ **9.94** rad/s.
- Under load only $24 - 7.84 = 16.16$ V drives speed: $16.16 \times 12.42/30 =$ **6.69** rad/s.
- $90° = 1.571$ rad: $t \geq 1.571/6.69 =$ **0.235** s.
- (c) $T_{valve} = KiN\eta = 0.08 \times 6 \times 30 \times 0.85 =$ **12.2** N $\cdot$ m, 1.5 times the load.
- $P = T_m\omega_m = 0.314 \times (6.69 \times 30) =$ **63** W.