UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

11.5 Forced vibration

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

KEY WORDS resonance • transmissibility • vibration isolation

Objective

Find the steady response of a machine to its own unbalance, the force it passes to the floor, the response at resonance, and the mount stiffness that isolates it.

1 Key knowledge and worked examples

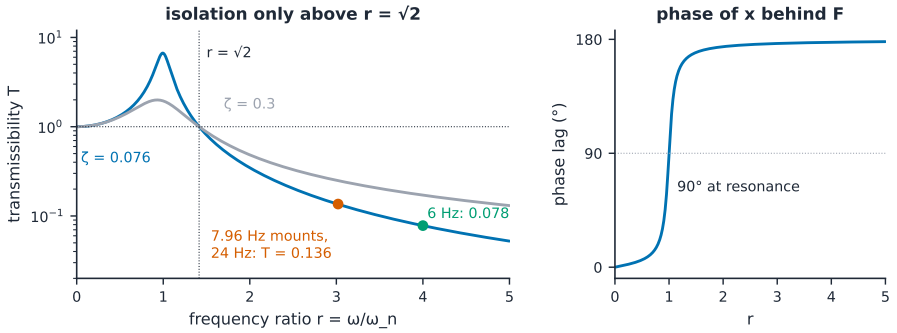
■ **Frequency ratio decides**

- Unbalance force $F_0 = me\,\omega^2$; frequency ratio $r = \omega/\omega_n$.
- Amplitude $X = \dfrac{F_0/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$; at **resonance** $(r = 1)$ the lag is 90° and, for unbalance, $X = me/(2\zeta m)$.
- **Transmissibility** $T = \sqrt{\dfrac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}}$: **vibration isolation** only above $r = \sqrt{2}$.

■ **Worked example. The present mounts**

$me = 1.2$ g · m at 1440 rpm ($\omega = 150.8$ rad/s), $f_n = 7.96$ Hz, $\zeta = 0.076$.

- $F_0 = me\,\omega^2 = \mathbf{27.3}$ N; $r = 24/7.96 = \mathbf{3.02}$.
- $T = \mathbf{0.136}$: 3.7 N reaches the floor; the pump moves 11 µm.
- 90% isolation needs $r \geq 3.40$; at $f_n = 6$ Hz, $k = 170.5$ kN/m and $T = 0.078$.



2 Practice

The compressor of sheet 11.4: $m = 200$ kg, $k = 400$ kN/m, $\omega_n = 44.72$ rad/s, $\zeta = 0.0567$. Its residual unbalance is $me = 2.0$ g · m and it runs at 1500 rpm.

2.1 Find ω, the unbalance force F_0 (one decimal, N) and the frequency ratio r (three decimals). [3]

2.2 Find the transmissibility (four decimals), the force reaching the floor (one decimal, N) and the steady amplitude (one decimal, µm). [5]

3 Exam-style questions

3.1 (a) Find the amplitude while the compressor passes through resonance during run-up (one decimal, µm) and say what the operator should do. (b) The site wants at most 5% transmissibility; for this ζ that needs $r \geq 4.88$. Find the highest allowed f_n (two decimals, Hz). (c) Choose $f_n = 5$ Hz: find the total and per-mount stiffness (one decimal, kN/m), the static deflection (one decimal, mm) and the floor force (two decimals, N). (d) Give one cost of the softer mounts. [9]

Solutions

2.1

- $\omega = 2\pi N/60 = 2\pi \times 1500/60 = 157.08$ rad/s.
- $F_0 = me\,\omega^2 = 0.002 \times 157.08^2 = \mathbf{49.3}$ N.
- $r = \omega/\omega_n = 157.08/44.72 = \mathbf{3.512}$.

2.2

- $2\zeta r = 0.398$; $1 - r^2 = -11.34$.
- $T = \sqrt{\dfrac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}} = \sqrt{\dfrac{1 + 0.398^2}{11.34^2 + 0.398^2}} = \mathbf{0.0949}$.
- Floor force $= TF_0 = 0.0949 \times 49.3 = \mathbf{4.7}$ N.
- $X = (F_0/k)/\sqrt{(1 - r^2)^2 + (2\zeta r)^2} = (49.3/400\,000)/11.35 = \mathbf{10.9}$ µm.

3.1

- (a) At $r = 1$: $X = me/(2\zeta m) = 0.002/(2 \times 0.0567 \times 200) = \mathbf{88.2}$ µm, eight times the running amplitude.
- Run up quickly through about 7 Hz so the resonance has no time to build.
- (b) $f_{n,max} = \omega/(2\pi r) = 157.08/(2\pi \times 4.88) = \mathbf{5.12}$ Hz.
- (c) $k = m(2\pi f_n)^2 = 200 \times (2\pi \times 5)^2 = \mathbf{197.4}$ kN/m, $\mathbf{49.3}$ kN/m per mount.
- $\delta_{st} = mg/k = 1962/197\,400 = \mathbf{9.9}$ mm.
- $r = 25/5 = 5.0$, $T = 0.0479$, floor force $= \mathbf{2.36}$ N.
- (d) The machine sits lower and moves more under pipe loads or start-up, so it needs flexible connections and travel stops.