UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

11.4 Free vibration

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

KEY WORDS natural frequency • damping ratio • underdamped • logarithmic decrement

Objective

Find the natural frequency and static deflection of a machine on its mounts, measure its damping from a decay record, and find the torsional frequency of two discs on a shaft.

1 Key knowledge and worked examples

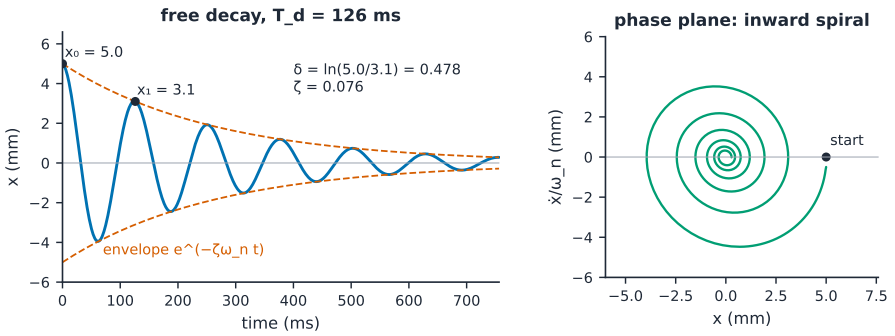
Mass, spring, damper

- $m\ddot{x}+c\dot{x}+kx=0$: **natural frequency** $\omega_n=\sqrt{k/m}$, $f_n=\omega_n/2\pi$, static deflection mg/k.
- **Damping ratio** $\zeta=c/(2\sqrt{km})$; **underdamped** below 1, ringing at $\omega_d=\omega_n\sqrt{1-\zeta^2}$.
- **Logarithmic decrement** $\delta=\ln(x_0/x_1)$ over one period; $\zeta=\delta/\sqrt{4\pi^2+\delta^2}$.
- Two discs on a shaft: $k_t=GJ_p/L$, $J_p=\pi d^4/32$, $\omega=\sqrt{k_t(J_1+J_2)/(J_1J_2)}$.

Worked example. The pump on its mounts

$m=120$ kg on four mounts of 75 kN/m, so $k=300$ kN/m.

- $\omega_n=\sqrt{k/m}=\sqrt{300\,000/120}=$ **50.0** rad/s, $f_n=$ **7.96** Hz.
- Peaks 5.0 and 3.1 mm: $\delta=\ln(5.0/3.1)=0.478$, $\zeta=$ **0.076**.
- $c=2\zeta\sqrt{km}=910$ N · s/m; $\omega_d=49.86$ rad/s.



2 Practice

2.1 A 200 kg compressor stands on four mounts of 100 kN/m each. Find ω_n (two decimals, rad/s), f_n (two decimals, Hz) and the static deflection (one decimal, mm). [4]

2.2 A tap test gives successive peaks of 4.0 mm and 2.8 mm one period apart. Find δ (four decimals), ζ (four decimals), c (nearest N · s/m) and ω_d (two decimals, rad/s). [5]

3 Exam-style questions

3.1 A steel shaft ($G = 80$ GPa), 40 mm in diameter and 0.8 m long, joins a flywheel ($J_1 = 1.0$ kg $\cdot$ m^2) to a fan ($J_2 = 0.25$ kg $\cdot$ m^2). (a) Find J_p and k_t (one decimal, kN $\cdot$ m/rad). (b) Find the torsional natural frequency in rad/s, Hz and rpm. (c) Locate the node measured from the flywheel, and say whether running at 1500 rpm is safe. [8]

Solutions

2.1

- $k = 4k_{mount} = 4 \times 100 = 400$ kN/m; $\omega_n = \sqrt{k/m} = \sqrt{400\,000/200} = \mathbf{44.72}$ rad/s.
- $f_n = \omega_n / 2\pi = \mathbf{7.12}$ Hz.
- $\delta_{st} = mg/k = 200 \times 9.81 / 400\,000 = \mathbf{4.9}$ mm.

2.2

- $\delta = \ln(x_0/x_1) = \ln(4.0/2.8) = \mathbf{0.3567}$.
- $\zeta = \delta / \sqrt{4\pi^2 + \delta^2} = 0.3567 / 6.293 = \mathbf{0.0567}$.
- $c = 2\zeta\sqrt{km} = 2 \times 0.0567 \times \sqrt{400\,000 \times 200} = \mathbf{1014}$ N · s/m.
- $\omega_d = \omega_n \sqrt{1 - \zeta^2} = \mathbf{44.65}$ rad/s, almost unchanged.

3.1

- (a) $J_p = \pi d^4 / 32 = \pi \times 0.04^4 / 32 = 2.513 \times 10^{-7}$ m^4.
- $k_t = G J_p / L = 80 \times 10^9 \times 2.513 \times 10^{-7} / 0.8 = \mathbf{25.1}$ kN · m/rad.
- (b) $\omega = \sqrt{k_t (J_1 + J_2) / (J_1 J_2)} = \sqrt{25\,133 \times 1.25 / 0.25} = \mathbf{354.5}$ rad/s.
- $f = 354.5 / 2\pi = \mathbf{56.4}$ Hz $= \mathbf{3385}$ rpm.
- (c) The discs swing in opposite directions with amplitudes in the ratio $J_2 : J_1$, so the node is $L J_2 / (J_1 + J_2) = 0.8 \times 0.25 / 1.25 = \mathbf{0.16}$ m from the flywheel.
- 1500 rpm is 44% of 3385 rpm, well clear of resonance; a 2nd-order excitation (3000 rpm) would come close.