UNIVERSITY MECHANICAL ENGINEERING • EXERCISE SHEET

11.3 Balancing

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS static balance • dynamic balance • two-plane balancing • reciprocating mass • primary force • secondary force

Objective

Balance a rotor with correction masses in two planes, find the shaking force it would otherwise exert, and find the primary and secondary forces of a reciprocating mass.

1 Key knowledge and worked examples

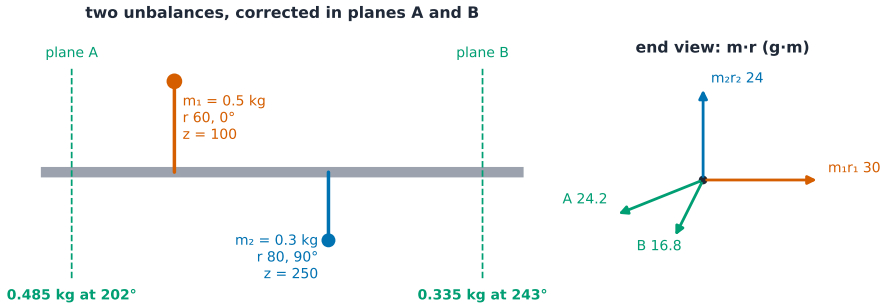
■ Force and moment, as vectors

- An unbalance mr at speed ω pulls with $mr\omega^2$.
- **Static balance** : $\sum m_i \mathbf{r}_i = 0$. **Dynamic balance** also needs $\sum m_i \mathbf{r}_i z_i = 0$: **two-plane balancing** .
- Moments about plane A give B: $m_B \mathbf{r}_B z_B = -\sum m_i \mathbf{r}_i z_i$; then $m_A \mathbf{r}_A = -\sum m_i \mathbf{r}_i - m_B \mathbf{r}_B$.
- **Reciprocating mass** : **primary force** $mr\omega^2 \cos\theta$, **secondary force** $mr\omega^2 \cos 2\theta / n$.

■ Worked example. The pump shaft

$m_1 = 0.5$ kg at 60 mm, 0°, $z = 100$ mm; $m_2 = 0.3$ kg at 80 mm, 90°, $z = 250$ mm; planes at $z = 0$ (A) and 400 mm (B), radius 50 mm.

- Moments about A: $m_B r_B = 16.8$ g · m, so **0.335** kg at **243**°.
- Forces: $m_A r_A = 24.2$ g · m, so **0.485** kg at **202**°.



2 Practice

2.1 A rotor carries $m_1 = 0.4$ kg at 50 mm, angle 0°, and $m_2 = 0.6$ kg at 40 mm, angle 120°. Find the resultant $\sum m\mathbf{r}$ as components and its size (two decimals, g · m), and the shaking force at 1200 rpm (one decimal, N). [4]

2.2 A 1.2 kg reciprocating mass runs on a 50 mm crank at 600 rpm with $n = l/r = 4$. Find the primary and secondary force amplitudes (one decimal, N) and the secondary frequency. [3]

3 Exam-style questions

3.1 In 2.1, m_1 sits at $z = 80$ mm and m_2 at $z = 220$ mm. Correction planes are at $z = 0$ (A) and $z = 300$ mm (B), with masses at 60 mm radius. (a) Find $\sum m\mathbf{r}z$ (components, g · m^2). (b) Find the correction in plane B and then in plane A: mass (three decimals, kg) and angle (one decimal, degrees). (c) Explain why one mass opposite the resultant of 2.1 would not do. [9]

Solutions

2.1

- $m_1\mathbf{r}_1 = (20, 0)$ g · m; $m_2\mathbf{r}_2 = 24(\cos 120°, \sin 120°) = (-12, 20.78)$ g · m.
- $\sum m\mathbf{r} = (8, 20.78)$ g · m, size **22.27** g · m.
- $\omega = 2\pi N/60 = 2\pi \times 1200/60 = 125.66$ rad/s; $F = mr\omega^2 = 0.02227 \times 125.66^2 =$ **351.7** N.

2.2

- $\omega = 62.83$ rad/s; primary $mr\omega^2 = 1.2 \times 0.05 \times 62.83^2 =$ **236.9** N.
- Secondary $mr\omega^2/n =$ **59.2** N, at twice the shaft frequency: 20 Hz.

3.1

- (a) $\sum m\mathbf{r}z = (20 \times 0.08 - 12 \times 0.22,\ 20.78 \times 0.22) = (-1.04, 4.573)$ g · m^2.
- (b) $m_B\mathbf{r}_B = -\sum m\mathbf{r}z/z_B = (3.467, -15.24)$ g · m, size 15.63 g · m.
- $m_B = 15.63/60 =$ **0.261** kg at **282.8**°.
- $m_A\mathbf{r}_A = -\sum m\mathbf{r} - m_B\mathbf{r}_B = (-11.47, -5.54)$ g · m, size 12.74 g · m.
- $m_A = 12.74/60 =$ **0.212** kg at **205.8**°.
- (c) A single mass cancels the force but sits in one plane, so the couple $\sum m\mathbf{r}z$ stays and rocks the shaft in its bearings: static balance only.